2nd INTERNATIONAL 2023 SYMPOSIUM ON CURRENT DEVELOPMENTS IN FUNDAMENTAL AND APPLIED MATHEMATICS SCIENCES

(ISCDFAMS 2023)

ABSTRACT AND FULL TEXT SYMPOSIUM BOOK

Assoc. Prof. Dr. Furkan YILDIRIM
Asst. Prof. Dr. Fatma SAĞSÖZ

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Editors:

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PRESENTATION

Dear Participants,

We are proud and happy to welcome you to the 2nd International Symposium on Current Developments in Fundamental and Applied Mathematics Sciences (ISCDFAMS 2023), hosted by Ankara Bilim University with the support of TÜBİTAK and in collaboration with Atatürk University, Ankara Bilim University, Ankara University, Ankara Yıldırım Beyazıt University, Gazi University, Erzurum Technical University, Erzincan Binali Yıldırım University, Ağrı İbrahim Çeçen University, and Gümüşhane University.

We need fundamental sciences to grasp what is going on in our universe and to better understand the world we live in. In this context, we aimed to contribute to the development and sharing of mathematical sciences and applications with the symposium we organized together and to bring together mathematical institutions, interdisciplinary researchers, mathematicians and statisticians from all over the world.

During our symposium, 100 papers were presented by valuable academics who participated face-to-face and online over 4 days. Participants came not only from Türkiye but also from more than ten countries, including Canada, Australia, Romania, the Czech Republic, Russia, Azerbaijan, Morocco, India, Algeria, Pakistan, Jordan, China, Iran, and Kyrgyzstan.

We express our thanks and gratitude to Prof. Dr. Ömer ÇOMAKLI, the rector of Atatürk University; Prof. Dr. Yavuz DEMİR, the rector of Ankara Science University; Prof. Dr. Necdet ÜNÜVAR, the rector of Ankara University; Prof. Dr. Ali Cengiz KÖSEOĞLU, the rector of Ankara Yıldırım Beyazıt Üniversity; Prof. Dr. Musa YILDIZ, the rector of Gazi University; Prof. Dr. Bülent ÇAKMAK, the rector of Erzurum Technical University; Prof. Dr. Akın LEVENT, the rector of Erzincan Binali Yıldırım University; Prof. Dr. Abdulhalik KARABULUT, the rector of Ağrı İbrahim Çeçen University and Prof. Dr. Halil İbrahim ZEYBEK, the rector of Gümüşhane University, for their support.

We also extend our gratitude to TÜBİTAK for its support in organizing our symposium.

Special thanks to Atatürk University Scientific Research Projects Coordinatorship, Corporate Communications Directorate, Computer Science Research and Application Center Directorate and Atatürk University Publishing House Directorate.

Finally, we appreciate the efforts of organizing committee and scientific committee members who contributed to the realization of this event, the invited speakers, and all the valuable scientists who participated with their papers.

Assoc. Prof. Dr. Furkan YILDIRIM

Symposium Chairman

TAKDİM

Saygıdeğer Katılımcılar,

Ankara Bilim Üniversitesi ev sahipliğinde, Atatürk Üniversitesi, Ankara Bilim Üniversitesi, Ankara Üniversitesi, Ankara Yıldırım Beyazıt Üniversitesi, Gazi Üniversitesi, Erzurum Teknik Üniversitesi, Erzincan Binali Yıldırım Üniversitesi, Ağrı İbrahim Çeçen Üniversitesi ve Gümüşhane Üniversitesi iş birliği ve TÜBİTAK desteği ile düzenlenen II. Uluslararası Temel ve Uygulamalı Matematik Bilimlerinde Güncel Gelişmeler Sempozyumu'nda sizleri ağırlamaktan gurur ve mutluluk duyuyoruz.

Evrenimizde yaşananları daha iyi kavrayabilmek ve yaşam sürdüğümüz dünyayı daha iyi tanıyabilmek için temel bilimlere ihtiyaç duymaktayız. Bizler de bu kapsamda el birliğiyle organize ettiğimiz sempozyum ile matematik bilimlerinin ve uygulamalarının gelişimine, paylaşımına katkıda bulunmayı ve dünyanın her yerinden matematik kuruluşlarını, disiplinler arası araştırmacıları, matematikçileri ve istatistikçileri bir araya getirmeyi hedefledik.

Sempozyumumuzda, Türkiye'nin yanı sıra Kanada, Avustralya, Romanya, Çek Cumhuriyeti, Rusya, Azerbaycan, Fas, Hindistan, Cezayir, Pakistan, Ürdün, Çin, İran, Kırgızistan gibi ondan fazla ülkeden katılan değerli akademisyenler tarafından 4 gün içerisinde yüz yüze ve çevrim içi olarak sunulan 100 bildiri yer almaktadır.

Sempozyumumuzun düzenlenmesinde desteklerinden dolayı Atatürk Üniversitesi Rektörü Sayın Prof. Dr. Ömer ÇOMAKLI' ya, Ankara Bilim Üniversitesi Rektörü Sayın Prof. Dr. Yavuz DEMİR' e, Ankara Üniversitesi Rektörü Sayın Prof. Dr. Necdet ÜNÜVAR' a, Ankara Yıldırım Beyazıt Üniversitesi Rektörü Sayın Prof. Dr. Ali Cengiz KÖSEOĞLU' na, Gazi Üniversitesi Rektörü Sayın Prof. Dr. Musa YILDIZ' a, Erzurum Teknik Üniversitesi Rektörü Sayın Prof. Dr. Bülent ÇAKMAK' a, Erzincan Binali Yıldırım Üniversitesi Rektörü Sayın Prof. Dr. Akın LEVENT' e, Ağrı İbrahim Çeçen Üniversitesi Rektörü Sayın Prof. Dr. Abdulhalik KARABULUT' a ve Gümüşhane Üniversitesi Rektörü Sayın Prof. Dr. Halil İbrahim ZEYBEK' e teşekkür ve şükranlarımızı sunarız.

Ayrıca sempozyumumuzun düzenlenmesinde desteğinden dolayı TÜBİTAK' a teşekkür ederiz.

Atatürk Üniversitesi Bilimsel Araştırma Projeleri Koordinatörlüğüne, üniversitemiz Kurumsal İletişim Direktörlüğüne, üniversitemiz Bilgisayar Bilimleri Araştırma ve Uygulama Merkezi Müdürlüğüne ve üniversitemiz Yayınevi Müdürlüğüne teşekkürlerimizi sunarız.

Son olarak bu etkinliğin gerçekleşmesinde emeği geçen düzenleme kurulu ve bilim kurulu üyelerine, davetli konuşmacılara ve bildirileriyle katılım sağlayan tüm değerli bilim insanlarına teşekkür ederiz.

Doç. Dr. Furkan YILDIRIM

Sempozyum Başkanı

OPENING SPEECH

OPENING SPEECH

Respected Rectors and Vice-Rectors, distinguished colleagues, esteemed participants and dear students,

It is with great pleasure and honor that I stand before you today as the member of the organizing commite of the "2nd International Symposium on Current Developments in Fundamental and Applied Mathematics Sciences." This symposium represents a significant collaboration among nine prestigious universities, a testament to the global significance of mathematics in today's world. In addition, our symposium has been entitled to receive the 2223-B project support of TUBITAK.

As we gather here in the beautiful capital city of Ankara, I am reminded of the profound impact mathematics has on our lives. It transcends borders and cultures, unifying us in our pursuit of knowledge and understanding. Mathematics is the universal language that allows us to explore, discover, and innovate, shaping our world in remarkable ways. Our symposium is a platform for sharing the latest advancements in both fundamental and applied mathematics. It's a testament to the dedication and hard work of countless researchers, educators, and students who have contributed to the field. Together, we celebrate not only the achievements but also the limitless potential that lies ahead. The theme of this symposium is a testament to the dynamic nature of mathematics. It's a subject that constantly evolves, adapting to the changing needs of society. Fundamental mathematics provides the solid foundation upon which all other branches of the discipline are built. It's the bedrock of our understanding, allowing us to explore the fundamental principles that govern the universe.

Applied mathematics, on the other hand, takes these principles and transforms them into practical solutions that influence our daily lives. Whether it's optimizing transportation systems, managing financial risks, or advancing medical research, applied mathematics plays a pivotal role in shaping our modern world. As we engage in discussions, presentations, and collaborations over the coming days, I encourage you to embrace the diversity of thought and ideas that this symposium offers. It is through this diversity that we find new perspectives, innovative solutions, and the potential to make real-world impact. I also want to take this moment to express my sincere gratitude to the nine universities that have come together to make this symposium a reality. Their support and partnership have been invaluable in creating this gathering of brilliant minds.

In closing, I would like to extend my warmest welcome to all our participants, both near and far. Your presence here enriches this symposium, and I hope you will find inspiration, knowledge, and lasting connections during your time in Ankara.

As we embark on this intellectual journey together, let us remember that mathematics is not merely a subject; a shared passion binds us as a community. Let us strive for excellence, encourage innovation, and contribute to the ever-expanding realm of mathematical sciences. Thank you for being a part of this excellent event. I look forward to the insightful discussions and collaborations that will undoubtedly arise during the symposium. I would like to thank the scientific committee, organizing committee, and all of the sponsors for their hard work in making this symposium possible. I would also like to thank the participants for choosing to spend your time here with us.

Thank you again for coming to the 2nd International Symposium on Current Developments in Fundamental and Applied Mathematics Sciences. I wish you a productive and enjoyable conference.

Prof. Dr. Kürşat Akbulut

Speaker on behalf of the Organizing Committee

KEYNOTE SPEECH

On Geometric and Physical Interpretations of Basic Concepts on a (Semi-Riemann) Manifold

Bayram Şahin

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Abstract: In the theory of manifolds, many notions are defined algebraically and properties are examined using algebraic operations. This prevents the geometric meaning of the concepts in the theory of manifolds and the importance of their physical applications to be understood except by experts in the field. In this talk, geometric and physical interpretations of the basic notions on a (semi-Riemannian) manifold will be given in a way that can be followed by non-experts in the field of manifold theory.

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INVITED SPEAKERS

A New Lift Construction of Connections in the Bundle of 2-jets

Arif Salimov and Narmina Gurbanova

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Abstract: Let $T^2(M_r)$ be the bundle of 2-jets, i.e. the tangent bundle of order 2 over C^∞-manifold M_r, $\dim T^2(M_r)=3r$ and let

$$(x^i,x^{\bar{i}},x^{\bar{\bar{i}}})=(x^i,x^{r+i},x^{2r+i}),\ x^i=x^i(t),\ x^{\bar{i}}=\frac{dx^i}{dt},\ x^{\bar{\bar{i}}}=\frac{1}{2}\frac{d^2x^i}{dt^2},\ t\in\mathbb{R},\ i=1,...,r$$

be an induced local coordinates in $T^2(M_r)$. Let $\tilde{\nabla}$ be a pure projectable connection on $T^2(M_r)$ with respect to Π, where Π is the regular structure naturally existing on $T^2(M_r)$. Firstly, we prove that the bundle $T^2(M_r)$ is a real modeling of $R(\varepsilon^2)$-holomorphic manifold $X_r(R(\varepsilon^2))$ (see [1]), where $R(\varepsilon^2)$ is the algebra of order 3 with a canonical basis $\{e_1,e_2,e_3\}=\{1,\varepsilon,\varepsilon^2\}$, $\varepsilon^3=0$. We would like to find a local expression of $\tilde{\nabla}$ on $T^2(M_r)$ which is corresponding to the $R(\varepsilon^2)$-holomorphic connection $\overset{*}{\nabla}$ on $X_r(R(\varepsilon^2))$.

The pure projectable connection $\tilde{\nabla}$ on $T^2(M_r)$ has components

$$(\tilde{\Gamma}^k_{\alpha\beta})=\begin{pmatrix}\Gamma^k_{ij}(x^m) & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix},\ (\tilde{\Gamma}^{\bar{k}}_{\alpha\beta})=\begin{pmatrix}x^t\partial_t\Gamma^k_{ij}+G^k_{ij}(x^m) & \Gamma^k_{ij}(x^m) & 0\\ \Gamma^k_{ij}(x^m) & 0 & 0\\ 0 & 0 & 0\end{pmatrix},$$

$$(\tilde{\Gamma}^{\bar{\bar{k}}}_{\alpha\beta})=\begin{pmatrix}x^{\bar{\bar{s}}}\partial_s\Gamma^k_{ij}+\frac{1}{2}x^{\bar{t}}x^{\bar{s}}\partial_t\partial_s\Gamma^k_{ij}+x^{\bar{s}}\partial_sG^k_{ij}+H^k_{ij} & x^{\bar{t}}\partial_t\Gamma^k_{ij}+G^k_{ij} & \Gamma^k_{ij}\\ x^{\bar{t}}\partial_t\Gamma^k_{ij}+G^k_{ij} & \Gamma^k_{ij} & 0\\ \Gamma^k_{ij} & 0 & 0\end{pmatrix}$$

with respect to the induced coordinates $(x^r,x^{\bar{r}},x^{\bar{\bar{r}}})$ in $T^2(M_r)$, where $H=(H^k_{ij}(x^1,...,x^r))$ and $G=(G^k_{ij}(x^1,...,x^r))$ are arbitrary (1,2)-tensor fields on M_r. If $H=G=0$, then we have $\tilde{\nabla}=\nabla^C$, where ∇^C is the complete lift of ∇ to $T^2(M_r)$ [2, p.338]. The connection $\tilde{\nabla}$ on $T^2(M_r)$ is called the deformed complete lift of ∇ to $T^2(M_r)$.

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Omega Invariant and Realizability of Graphs

Ismail Naci Cangul[1]

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Abstract: Recently defined as a useful alternative to Euler characteristic of a given graph, the omega invariant of a graph was introduced four years ago to determine several combinatoric, algebraic and topological properties of all realizations of a given degree sequence or of any particular graph. It is very much related to the Euler characteristic and the cyclomatic number. It helps one to find many algebraic, graph theoretical, number theoretical, topological and combinatorial properties of all the realizations of the given degree sequence including cyclicity, connectivity, numbers of components, multiple edges, loops, cycles, chords, pendant and support vertices, etc. Since 2019, several applications of this invariant have been found. In this work, we shall recall the omega invariant together with some fundamental combinatoric properties and try to solve an open problem. This problem is to find the number of realizations of a given degree sequence. Here we shall give some partial results on this problem.

Keywords: Graph characteristic, degree sequence, omega invariant, realizability.

1. Main results

We take G=(V,E) as a graph with |V(G)| = n vertices and |E(G)| = m edges. m and n are respectively called the order and size of G. The degree of a vertex v in V(G) is denoted by d_v. A vertex with degree one is called a pendant vertex. Written with multiplicities, a degree sequence in general is written as $DS(G) = \{0^{(a_0)}, 1^{(a_1)}, 2^{(a_2)}, 3^{(a_3)}, \ldots, \Delta^{(a_\Delta)}\}$. For a set D of non-decreasing non-negative integers, we say that G is a realization of D if the degree sequence of G is equal to D.

The number a_1 of pendant vertices of a tree T is given by $a_1 = 2 + a_3 + 2a_4 + 3a_5 + 4a_6 + \ldots + (\Delta-2)a_\Delta$ where Δ is the largest vertex degree in T and a_i denotes the number of vertices of degree i. This equation can be rearranged as $a_3 + 2a_4 + 3a_5 + 4a_6 + \ldots + (\Delta-2)a_\Delta - a_1 = -2$. The authors observed in [1] that the left hand side of this equation is a graph invariant for all graphs with the same degree sequence D and denoted it by $\Omega(D)$. This invariant has a big potential to give structural and combinatorial information about all realizations of a degree sequence. For any graph G, $\Omega(G) = 2(m-n)$, $\Omega(G)$ is additive and if there are c components, then the number r of closed regions is given by $r = \Omega(G)/2 + c$, [1,2]. Also for any graph, we have $\Omega(G) = 2(r- \chi(G))$.

This implies that $c \geq -\Omega(G)/2$ and equivalently, $c \geq n-m$ giving a lower bound for the number of components of all realizations of a given degree sequence. We can improve this bound as follows:

Theorem 1. The minimum number of components for all realizations of D is $c_{min} = \max\{1, -\Omega(D)/2\}$.

In [2], the maximum number of components for all realizations of D is given by the sum $c_{max} = \sum_{di \text{ is even}} ai + \sum_{di \text{ is odd}} ai$. Therefore, for a given degree sequence D, we know the minimum and maximum number of components of all the realizations of D.

In this work, we try to calculate the number of realizations of a given degree sequence. Let us denote the total number of realizations of D with N(D) and the number of realizations of D with c components by $N_D(c)$. The algorithm we follow is as follows:

Algorithm 1. Step 1. Calculate c_{max}. Then find $N_D(c_{max})$ by drawing the possible realizations with c_{max} components.

Step 2. Find the ways to join two components in all possible ways and find $N_D(c_{max}-1)$.

Step 3. Eliminate the isomorphic realizations.

Step 4. Repeat Steps 2 and 3 to find $N_D(c_{max}-2)$, $N_D(c_{max}-3)$, $N_D(c_{max}-4)$, …, $N_D(c_{min})$.

Step 5. Find the number $N(D) = N_D(c_{max}) + N_D(c_{max}-1) + N_D(c_{max}-2) + \ldots + N_D(c_{min})$.

The first step is the easy part. The hardest part of this algorithm is to eliminate the isomorphic realizations. We shall try to give some ways to achive this. The calculations will start with the selection of degree sequences. As the vertices of even degree are easier to transfer onto other components, we shall start with those degree sequences having only even vertex degrees. In this case, clearly $N_D(c_{max}) = 1$. We shall prove the following as the first step:

Theorem 2. Let D be a degree sequence having all entries as positive even integers. Let there be k different entries and let there be t entries with multiplicity at least 2 where $t \leq k$. Then

$$N_D(c_{max}-1) = \binom{k}{2} + t.$$

Next problem following the above will be to obtain similar formulae for the other summands. As the second step, we shall add some odd vertex degrees to D and make similar calculations which seem to be more complicated and detailed and left as an open problem.

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Certain Saigo Type Fractional Integral Inequalities and Their q-analogues

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Abstract: We summarize our research work by mentioning that all the results derived in this paper are novel and important. Firstly, we have established ceratin inequalities involving Saigo type fractional integral operator and derived some special cases of it. Then we have derived q-analogues of the inequalities involving Saigo type fractional integral operator that means certain q-integral inequalities. Some special cases of q-integral inequalities also derived.

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Approximations of Surfaces Along Curves

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Abstract: In this study, we discussed the approximations of surfaces for parallel surfaces. If special curves are selected on the surfaces, what will be the relations with the approach surface?

Keywords: Parallel surfaces, curves on surfaces**,** developable surfaces.

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Self-Symmetries in Descriptively Near Envelopes

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Abstract: Signal that is an envelope that modulates the waveform of h(t) and m(t) = x(t)h(t) is a modulated input signal in an envelope. From a topology perspective, we consider envelopes that are contractible open sets good covers of bounded oscillating wave forms and which are examples of the geometric Lusternik-Schnirel'mann Montejano categories (gcats). An application (with examples) of gcats in LTD systems will be given in terms of

(1) closeness of envelopes on self-similar video lumens signals and

(2) closeness of envelopes on self-similar signals from AM radio receivers.

Keywords: Signal, self-symmetries, LTD systems.

Some Conjectures About Number Theory Triangles Linked to Merge Sorting

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Abstract: This paper explores some number theory issues arising from a matrix developed for the cascade sorting of tapes. The aim here is to use the array to consider conjectures, especially in relating to ideas for mathematics students in learning to guess shrewdly. The triangles which come from the matrices suggest further developments. The problems are connected with some computer science connections with number theory, but the paper is not about computational sorting *per se*, but some background diagrams are included. The reasons for the appearances of the sequences identified in the arrays and related to Sloane's *Encyclopedia of Integer Sequences* are a suggestion for further study.

Keywords: Merge sorting, generalized *s*-step Fibonacci sequences and *s*-step Pascal triangles, difference operators, stair-step diagonal counting.

Geodesic Mappings and Their Generalization

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Abstract: The important part of the contemporary differential is a study of mappings that map geodesics onto special curves, i.e., geodesics, almost geodesics, analytic curves, etc. The theories of these mappings have applications in theoretical physics for modeling gravitational, electromagnetic, and other fields.

Geodesic mappings are mappings that map any geodesic onto geodesic. Let $A_n = (M, \nabla)$ and $\bar{A}_n = (\bar{M}, \bar{\nabla})$ be affine spaces, where M and $\bar{M}$ are n-dimensional manifolds, ∇ and $\bar{\nabla}$ are torsion-free affine connections. It is known [4] that A_n admits geodesic mapping onto $\bar{A}_n$ if and only if the Levi-Civita equations

$$\bar{\nabla}_X Y = \nabla_X Y + \psi(X)Y + \psi(Y)X$$

hold for any tangent fields X, Y and where ψ is a linear differential form, ∇ and $\bar{\nabla}$ are the Levi-Civita connections. We can suppose that $\bar{M} = M$. If ψ is vanishing then the mapping f is *affine* or *trivially geodesic*.

It was proved that geodesic mappings of geodesic mappings of (pseudo-)Riemannian spaces onto (pseudo-) Riemannian spaces preserve the differentiability class of metric [2]. The similar holds for geodesic mappings of spaces with affine connection onto (pseudo-)Riemannian spaces [3]. Furthermore, it was also proved that all affine spaces are geodesically equivalent to equiaffine spaces [10]. Mikeš and Berezovski [5] found a criterion (equivalent to the Levi-Civita equations) for the existence of geodesic mapping of equiaffine space onto (pseudo-)Riemannian space. The criterion was in the form of linear partial differential equations in covariant derivatives. These equations hold globally and also locally. See [4].

A special part is also dedicated to the existence of so-called geodesic bifurcation. This term describes the situation where there exist more geodesics passing through the given point in the given direction, the case where the condition of geodesic uniqueness is not fulfilled. The example of geodesic bifurcation was given on the surface of revolution. Moreover, based on

this example, another example of closed geodesic was given as well as an example of geodesic bifurcation of product spaces [9].

Next, S.G. Leiko defined so-called rotary mapping. Rotary mappings are mappings that map any geodesic onto an isoperimetric extremal of rotation. Isoperimetric extremal of rotation is a special curve that is an extremal of functional of length and functional of rotation. These curves also have an interpretation in physics - they are trajectories of particles with a spin. We generalized this term and found a counterexample of spaces that admit rotary mappings [6]. We have found a general solution for the basic equations of surfaces of revolution.

Finally, we mention the basics of the projective transformation and holomorphically projective transformations (of Kähler manifolds) theory. In the end, we also briefly mention the basics of *F*-planar mappings, their definitions, and certain results of these theories [1, 4, 7, 8].

Keywords: Geodesic mapping, geodesic, Riemannian spaces.

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ABSTRACTS

Existence and Uniqueness Results of a Lasota-Wazewska Model with an Iterative Term

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Abstract: In this study, we consider a new Lasota-Wazewska type delay differential equation with an iterative term and a nonlinear harvesting. We establish the existence of at least one positive periodic solution by the Schauder's fixed point theorem with the Green's functions method. Moreover, we use the Banach contraction principle for proving the existence of a unique solution that depends continuously on parameters. Our results are new and complement previous studies.

Keywords: Delay differential equation, fixed point theory, positive periodic solution.

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Examination of the Existence of Coding on Violin Strings

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Abstract: In this study, a new coding and decoding algorithm was created by using the strings on the violin and notes on the frets corresponding to each string, bringing a new perspective to coding. In this method, we obtain different keys and messages. The purpose of this process is not only to increase the reliability of information security technology, but also to provide the ability to verify information at a high rate.

Keywords: Violin strings, Coding theory, Binary codes, Encryption and Decryption.

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Phase Type Distribution Queues

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Abstract: The PH/M/1 queue is a queuing system used to model the behavior of a single-server queue with arrivals following a general phase-type distribution (PH) and service times being exponentially distributed. The PH/M/1 queue is a powerful mathematical model that combines the flexibility of phase-type service time distributions with the simplicity of exponential inter-arrival times. This enables the analysis and study of various queuing systems in practical applications, such as telecommunications networks, computer systems, and service-oriented industries. The PH/M/1 queue allows researchers and analysts to derive important performance metrics, such as average queue length, average waiting time, and server utilization, to optimize system performance and resource allocation.

Keywords: Phase Type Distribution, PH Queue, Performance Analysis, Markov chain, single server

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Existence Results for Second Order Functional Differential Inclusions

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Abstract: Our aim in this work is to study the existence of solutions of second order for functional differential inclusions with state-dependent delay. We use *Mönch* 's fixed point theorem for the existence of solutions and the concept of measures of noncompactness.

Keywords: Functional differential inclusion, mild solution, infinite delay, state-dependent delay, fixed point, cosine function of operator, measure of noncompactness.

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Some Characterizations on Hypersurfaces of Riemannian Product Manifolds

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Abstract: The theory of product manifolds has important physical and geometrical aspects. There are also nice applications of product manifolds in Kaluza–Klein theory, brane theory and gauge theory [1-3]. Locally product manifolds were first introduced by S. Tachibana [4]. Hypersurfaces of almost product manifolds were initially investigated by S. Deshmukh, A. Sharfuddin and S. I. Husain in [5]. Some of the main formulas for hypersurfaces of almost product manifolds were obtained by T. Adati in [6]. The interesting aspect of examining these hypersurfaces is that they have a para-contact structure that is reduced from the product structure. Many relations can be obtained by using the properties provided by the para-contact and the product structure.

The main purpose of the present paper is to continue this work. For this purpose, some of the main formulas and relations on hypersurfaces of a Riemannian product manifold are presented. Obtained results are discussed on hypersurfaces admitting para-contact structures.

Keywords: Product manifolds, hypersurfaces**,** para-contact structure.

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Dual Number Sequences: Quantum Calculus Approach with Applications

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Abstract: In this present paper, we give a detailed study of a new generation of dual number sequences with components including quantum integers. Also some fundamental identities are, such as D'ocagnes, Cassini, Catalan, and Binet formulas and relations of the q-dual number sequences. After that, a new dual vector called q-dual Fibonacci vector is defined and then some properties of this vector apply to various properties of geometry which are not generally known in the geometry of dual space.

Keywords: Quantum calculus, q-dual Fibonacci vector, q-dual number sequences.

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On the Robustness of the Trajectories of Control Systems with Respect to Fast Consumption of the Control Resource

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Abstract: In this work robustness of the system's trajectories with respect to the fast consumption of the control resource is considered. The behavior of the system by the Urysohn type integral equation is described which is nonlinear with respect to the state vector and is affine with respect to the control vector. The control functions of the system are chosen from the closed ball centered at the origin with radius r in the space $L_q(\Omega; R^m)$, $q \in (1, \infty)$. This kind of control functions arise in the case when the control resource is exhausted by consumption, such as, energy, fuel, finance, food, etc. (see, e.g., [1]). Under appropriate conditions the robustness of continuous and p-integrable $(p^{-1} + q^{-1} = 1)$ trajectories of the system with respect to the fast consumption of the remaining control resource is discussed. It is proved that consuming of the remaining control resource in the domain with a sufficiently small Lebesgue measure, generates an insignificant deviation of the trajectory (see, [2]). The proved theorem allows to state that for a significant change in the trajectory of the system, it is advisable to avoid the aggressive spending of the control resource and to consume the control resource in economy mode, i.e. to consume the control resource in small portions.

Keywords: Control system, integral equation, robustness

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Investigation of Stresses Occurring in a Circular Disk Consisting of Different Annular Materials by Mathematic Modeling

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Abstract: In this study, the thermal stresses occurring for a multilayer thin annular disk were analytically analyzed. It is quite possible to come across studies related to disks in the literature. However, the materials used in this study are different from other studies as a whole. Recently, due to the increase in the use of machine parts in a very high temperature environment, the study of the thermo-mechanical behavior of temperature-dependent materials has become important. The results obtained at the end of this study have been shared with the literature and presented with graphs. In the analytical solution, reference is made to Kirchhoff's variable transformation to bring the nonlinear heat conduction equation into linear form. At the end of the analysis, it was observed that the thermal tensile strength of Boron carbide (B4C) with a high modulus of elasticity is higher than Silicon Nitrite (SI3N4), while Silicon Nitrite (SI3N4) is higher than Carbon Glass fiber. As a result, it is thought that similar results may occur.

Keywords: Mathematical formulation, young modulus, annular disk

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Teaching with Dynamic Mathematics Software Geogebra: Examining Postgraduate Theses

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Abstract: Among the technical aids that can be used in the teaching and learning process of mathematics education, there are two important software. These are computer algebra systems (CAS) and dynamic geometry software (DGS). CAS is software that can provide precise results when performing operations with numbers, symbols, algebraic expressions, and algebraic objects. DGS can visually represent geometric structures and the relationships between these structures. In addition, these softwares provide the ability to move and drag the structure and measure the shapes (Hohenwarter & Jones, 2007). In the early 2000s, GeoGebra software was designed to offer both CAS and DGS capabilities. Because this software combines two separate structures and presents them together, it was named dynamic mathematics software by its developer in a separate category (Doğan, 2013; Hohenwarter & Jones, 2007; Hohenwarter & Lavicza, 2007). It is considered that it is important to study the dissertations related to GeoGebra in Türkiye and to illustrate the current situation in order to provide guidance to researchers who are working in this field or will work in this new field. In this context, the aim of this study is to analyze the postgraduate theses related to GeoGebra in Türkiye. In the light of the findings, the similarities and differences among the studies have been highlighted and the current situation for the researchers has been presented. In this research, answers to the following questions were sought. In postgraduate theses on GeoGebra in the field of mathematics education in Türkiye;

- Which goals are aimed to be achieved?
- Which mathematical subjects were preferred?
- What is their distribution over the years?
- What is the distribution by degree programme (master's-doctorate)?
- What are the research patterns?
- What are the sample profiles?
- Which data collection instruments were used?
- What are the methods of data analysis?

In this study, the method of document analysis, which belongs to qualitative research methods, was used. Document analysis is a systematic method of studying and evaluating all printed and

visual materials and documents (Kıral, 2020). Document analysis allows systematic review of previously published works within the framework of certain criteria. The results of the study will be incorporated into the symposium.

Keywords: Mathematics education, GeoGebra, Document analysis

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On Some Recursive Methods in Matrices Operations

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Abstract: In this work we discuss some recursive methods for operations on matrices, in particular the Divide and Conquer (D&C) strategy, which solves the problem by dividing it into sub-problems, which are themselves smaller instances of the same type of original problem. We will see how this technique creates a numerical recursive algorithm to compute the inverse of square matrices and solve linear systems using Shur's complement, Cholsky's complement and LU decomposition.

Keywords: Recursive method, Divide and Conquer, Schur complement, matrix decomposition, matrix inversion.

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Algebraic and Differential Identities on Rings and Algebras

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Abstract: The purpose of this research is to find an extension of the renowned Chernoff theorem [Journal Funct. Anal., 2 (1973)] on standard operator algebra. Infact, we prove the following result: Let H be a real (or complex) Banach space and $L(H)$ be the algebra of bounded linear operators on H. Let $A(H) \subset L(H)$ be a standard operator algebra. Suppose that D from $A(H)$ $to\ L(H)$ is a linear mapping satisfying the relation $D(A^nB^n) = D(A^n)\,B^n + A^n\,D(B^n)$ for all $A, B\ in\ A(H)$. Then D is a linear derivation on $A(H)$. In particular, D is continuous. Moreover, we discuss similar kind of identities on generalized left derivations on rings and furnish an application in context with Banach algebra.

Keywords: Semiprime ring, Banach space, standard operator algebra, derivation.

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Dual Vectors with Jacobsthal and Jacobsthal-Lucas Number Sequences

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Abstract: In this paper we introduce the dual Jacobsthal and the dual Jacobsthal-Lucas number sequences using dual numbers. Also given some formulas, facts, and properties about these number sequences. In addition, a new dual vector called the dual Jacobsthal vector is presented and then some properties of this vector apply to various properties of geometry which are not generally known in the geometry of dual space. In the future, researchers may examine many more identities of the dual numbers with dual Jacobsthal and dual Jacobsthal-Lucas number sequences. Consequently, these number sequences can be used in interdisciplinary studies.

Keywords: Dual Jacobsthal number sequences, Dual Jacobsthal-Lucas number sequences, Dual angle

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Properties and Applications of Bipolar Fuzzy Sets

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Abstract: In this paper, firstly, bipolar fuzzy sets and their properties found in the literature are examined. The definition of interval-valued fuzzy sets is given. In this paper, firstly, bipolar fuzzy sets and their properties found in the literature are examined. The definition of interval-valued fuzzy sets is given. Then, the definition and applications of bipolar soft sets are made. The positive degree and negative degree of the corresponding property of bipolar fuzzy sets are given in the relevant explanations, and their importance is emphasized. Additionally, operations between the elements of bipolar fuzzy sets are mentioned. An entropy measure has been defined in bipolar fuzzy sets. A new approach has been introduced to the decision process in bipolar fuzzy soft sets and an algorithm has been created. By following the algorithm, some analysis and detection are obtained. Entropy measures and parameter values are evaluated together and tabulated. As a result, the best few alternative choices are determined in the decision process with the given algorithm. Since the algorithm steps and application are practical, it is recommended as a method to be applied in many data groups.

Keywords: Fuzzy set, bipolar fuzzy set, bipolar soft set, entropy, decision making.

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Some Notes on Clairaut Conformal Riemannian Submersions

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Abstract: In this study, we define Clairaut conformal submersions whose total manifolds admit a Ricci soliton and provide a non-trivial example of such Clairaut conformal submersions. We firstly calculate scalar curvature and Ricci tensors of total manifolds of Clairaut conformal submersions and provide necessary conditions for the fibres of such Clairaut conformal submersions to be almost Ricci solitons and Einstein.

Keywords: Conformal submersion, Clairaut conformal submersion, Ricci soliton, Riemannian manifold, Riemannian submersion.

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A Variational Approach to a Constrained Hyperbolic Kirchhoff Type Problem

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Abstract: In this paper, we present a hyperbolic Kirchhoff-type problem with volume constraint. The problem includes a nonlocal term corresponding to a volume constraint. We are concerned about the construction of a solution to a hyperbolic Kirchhoff-type problem. The method that we used here to accomplish this aim is called the discrete Morse Fow (DMF) of hyperbolic type. Which consists of replacing the original problem with a sequence of minimization problems at discrete time levels which is a perfect choice for the nonlocal term.

Keywords: Hyperbolic Kirchhoff partial differential equation, Volume constraint, Discrete Morse Fow, Minimization problem.

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Some Bounds for the Topological Indices on Fuzzy Graphs

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Abstract: Let G=(V,E) be a graph with n vertices and m edges. A fuzzy graph G=(α,β) is a pair of functions which α is from V to [0,1] and β is from V×V to [0,1] with a fuzzy relation on α such that $\beta(u,v) \leq \min\{\alpha(u),\alpha(u)\}$ for all u,v in V. The fuzzy Randic index is an extension of the Randic index, which is a topological index that characterizes the molecular branching and this index is defined by Kalathian et al. in 2019. Şeyran and Kaya introduced the Fuzzy Randic energy (FRE) and Fuzzy Randic Estrada index (FREE) in 2023 using the eigenvalues of the fuzzy Randic matrix of fuzzy graphs. In this study, new bounds for Fuzzy Randic energy and Fuzzy Randic Estrada index are given.

Keywords: Fuzzy graphs, fuzzy Randic energy, fuzzy Randic Estrada index.

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Tame and Wild Automorphisms on Free Leibniz Algebras

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Abstract: Consider a free Leibniz algebra denoted as F with a finite rank of n, where n is greater than or equal to 2, and defined over a characteristic-0 field K. This investigation centers on the Jacobian matrices linked to specific endomorphisms within F. The core inquiry revolves around the Jacobian problem within F, aiming to discern automorphisms whose associated Jacobian matrices are invertible. Notably, it remains an open question whether free Leibniz algebras with a rank exceeding two can harbor what are known as "wild" automorphisms.

Our study offers a definitive criterion for the classification of automorphisms in F as either "tame" or "nontame," a crucial distinction primarily reliant on the determinant of the respective Jacobian matrices. This criterion provides clarity and precision in distinguishing between these categories, shedding light on the structural properties of automorphisms within F.

Moreover, our findings extend beyond the confines of F and find applicability in the realm of free metabelian Leibniz algebras.

Keywords: Leibniz algebra, Jacobian matrices, tame automorphism.

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A Note on Hopf's Conjecture

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Abstract: The classical *Hopf conjecture* asking whether an even-dimensional positive curvature Riemannian manifold (M, g) has positive Euler characteristic $\aleph(M)$ is maybe the most popular open question in global Riemannian geometry and perhaps even in the entire field of Riemannian geometry [1]. Despite the elegance and relevance of Hopf's hypothesis, it remains open in dimensions 6 and higher.

The main result of our report is a partial answer to Hopf's question. It is formulated as a version of the well-known *differentiable sphere theorem* (see [2]). Namely, we prove the following proposition: An n-dimensional $(n \geq 2)$ compact Riemannian manifold (M, g) without boundary and of positive sectional curvature is diffeomorphic to a spherical space form $\mathbb{S}^n/\Gamma$ if its Ricci curvature satisfies the strict inequality $Ric < n\,K_{min}$ where K_{min} is the minimum of the sectional curvature. Furthermore, if (M, g) is a compact simply connected Riemannian manifold without boundary and of positive sectional curvature and if the Ricci curvature satisfies the non-strict inequality $Ric \leq n\,K_{min}$, then (M, g) is diffeomorphic to a Riemannian locally symmetric space.

Keywords: Hopf conjecture, differentiable sphere theorem, spherical space form.

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On the Generators of $\mathbb{F}_2\mathbb{F}_4$-skew Cyclic Codes

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Abstract: Skew cyclic codes are one of the important family of codes in coding theory which are actually generalization of classical cyclic codes. In this work, we introduce and study the algebraic structure of skew cyclic codes over the mixed alphabet $\mathbb{F}_2^r \times \mathbb{F}_4^s$ where $\mathbb{F}_2$ and $\mathbb{F}_4$ are finite fields of 2 and 4 elements, respectively with non-negative integers *r* and *s*. We first determine the generator polynomials of $\mathbb{F}_2\mathbb{F}_4$-skew cyclic codes. Further, we relate skew cyclic codes over $\mathbb{F}_2^r \times \mathbb{F}_4^s$ to binary linear codes by defining a linear Gray map. We also investigate the duals of skew cyclic codes over this mixed alphabet. We show that the dual of any $\mathbb{F}_2\mathbb{F}_4$-skew cyclic code is another $\mathbb{F}_2\mathbb{F}_4$-skew cyclic code. As an application of our work, we provide several examples of $\mathbb{F}_2\mathbb{F}_4$-skew cyclic codes whose binary images have optimal parameters under the Gray mapping. We also consider the quantum codes over $\mathbb{F}_2^r \times \mathbb{F}_4^s$. By using the well-known construction method of quantum codes (CSS), we obtain examples of quantum codes with good parameters.

Keywords: Skew cyclic codes, $\mathbb{F}_2\mathbb{F}_4$, optimal codes, generator polynomials.

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Hyperbolic-Hyperbolic Singular Perturbation for Aquasilinear Problem with a Nonlinear Dissipative Term

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Abstract: In this work, we will deal with the global existence of the perturbation problem

$$u_{tt}-(\varepsilon+m(\|\nabla u\|^2))\Delta u+\delta u_t+f(u)=0, t>0, x\in\Omega$$

where $\delta>0$, ε is a parameter with $0<\varepsilon\leq 1$ and m a locally Lipschitz function, with $m(0)=0$ and $m(r)>0$ close to 0, and $f(u)u\geq 0$. We study the asymptotic behaviour of the solution $u_\varepsilon(t)$.

Keywords: Quasilinear, hyperbolic-hyperbolic singular perturbations, decay estimate, asymptotic behaviour.

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On the mock-Chebyshev Polynomial Interpolation

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Abstract: Polynomial interpolation with equidistant nodes is notoriously unreliable due to the Runge phenomenon and is also numerically ill-conditioned. The general recommendation is to use the highly non-uniform Chebyshev nodes [2]. But, during an observation or measurement, the data are generally obtained only at equally spaced points. When we aim to obtain an approximation with polynomial interpolation using equispaced nodes, one of the best strategies is the use of the lately developed mock-Chebyshev points in order to avoid the Runge phenomenon and to get similarly better results as in case of using Chebyshev-Lobatto points [1, 4]. Mock-Chebyshev points asymptotically follow the Chebyshev distribution, and they are selected from a large set of equispaced nodes. A disadvantage here is that the computational cost is higher and there are not many studies in the existing literature. The algorithms proposed in the recent papers [3, 4] provide robust methods for computing the mock-Chebyshev nodes for a given set of (n+1) Chebyshev-Lobatto points. In this study, by using the symmetry property of the mock-Chebyshev nodes, we propose modifications of the algorithms by reducing the computational costs by half. We also compare timings of the methods for the computation of the mock-Chebyshev nodes.

Keywords: Polynomial interpolation, Runge phenomenon, Mock-Chebyshev points.

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New Jacobi Wavelets' Kernel for Support Vector Machines

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Abstract : The first kernels investigated for support vector machines (SVM) problems were polynomial, Sigmoid, and radial basis kernels. Since the major task of the SVM approach lies in the selection of its kernel, several kernels were introduced and studied in the literature. Recently, considerable emphasis has been devoted to the construction based on orthogonal polynomial concepts. The underlying reason for this kernel type's success is the orthogonality property, which reduces the redundancy of the original data and makes it able to extract more discriminative features. Inheriting this property, kernels based on wavelets have great satisfaction in practice. The concept of combining orthogonal polynomial and wavelet theories led to the construction of novel wavelets such as the Legendre wavelets. Kernels based on this form of wavelet have never been introduced in the context of SVM before due to a continuity issue. In this work, we propose the new Jacobi wavelets family, which solves the continuity problem. We afterward developed a new kernel based on these new wavelets, where Legendre and Chebyshev wavelets represent specific cases, demonstrating the overall framework of our kernel. Although these Jacobi wavelets are not orthogonal, they form a wavelet frame, so we use frame theory to determine the Reproducing Kernel Hilbert Space (RKHS) of this kernel, and then we confirm that this new kernel has separation features. In two examples, we make a geometric comparison in two-dimensional space between our novel kernel and other kernels to confirm the performance of our approach. Experiment findings demonstrate that this kernel can compete with other kernels in terms of performance.

Keywords: Jacobi wavelets, Jacobi kernel, frame theory, Support vector machines.

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Regional Controllability of Fractional-order Dynamical System with Time-Varying Delays in the State

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Abstract: In this work, the regional controllability problem for fractional-order dynamical systems with time-varying delays in the state was considered using Caputo's standard derivative, in order to prove their regional exact and weak controllability. And also Depending on the Hilbert uniqueness method, we concentrate on the determination of control achieving regional controllability with minimum energy. In order to establish the credibility of our key findings, we examine two instances of fractional partial differential systems as illustrations-namely, the zonal actuator and the pointwise

actuator.

Keywords: Regional controllability, Fractional-order systems, Caputo derivatives, Time varying delays in the state, Optimal control, Minimum energy

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Biharmonic Sections

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Abstract: Let (M, g) be a Riemannian manifold. A vector field X on (M, g) is a section of the tangent bundle T M, the vector field X can be considered as a map between (M, g) and (T M, gS), i.e. X: (M, g) → (T M, gS), where T M carries the Sasaki metric gS induced from g. In this study, we introduce a class of vector fields on a Riemannian manifold which enlarges the class of harmonic vector fields (harmonic sections), called biharmonic sections. We prove that if M is compact, then X is biharmonic section if and only if X is parallel. In contrast with the harmonic case (see [2]), we show that the notions" biharmonic vector fields" (see [5]) and" biharmonic sections" are not equivalent. We prove that the left invariant vector fields on unimodular Lie groups of dimension three are biharmonic sections if and only if they are harmonic sections. Several examples are presented.

Keywords: Tangent bundle, harmonic section**,** biharmonic section.

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Impulsive Generalized Proportional Riemann-Liouville Fractional Differential Equations: Stability Concepts

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Abstract: One of the main properties studied in the qualitative theory of differential equations is the stability of solutions. There are various types of differential equations with different initial or boundary value conditions. Recently, several types of fractional derivatives are defined, studied and applied to model the dynamics or real processes and phenomena. Stability of fractional order systems is quite recent. There are several approaches in the literature to study stability, one of which is the Lyapunov approach. However the Lyapunov approach to fractional differential equations causes many difficulties. We study differential equations with generalized proportional fractional derivative of Riemann0Liouville type. Also, impulses are involved in the equation. The applied type of fractional derivative requires a new definitions of stability excluding the initial point as well as the impulsive times. Comparison results are presented and sufficient conditions for stability are given. Examples are presented to illustrate the theory**.**

Keywords: fractional differential equations, impulses, generalized proportional Riemann-Liouville fractional derivative, stability, Lyapunov functions.

Acknowledgement: The study is supported by Bulgarian National Science Fund under Project KP-06-N32/7.

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Obstacle Two-Phase Problems with Hardy Potential

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Abstract: Given $\Omega \subset R^n$ be a bounded domain, $n \geq 2$, containing the origin and with smooth boundary $\partial\Omega$. In the current paper, we are interested in the existence and the multiplicity of solutions to the following singular unilateral double phase problem:

$$-div(|\nabla u|^{p-2}\nabla u + \theta(x)|\nabla u|^{q-2}\nabla u) = \eta f(x,u) + \theta \frac{|u|^{p-2}u}{|x|^{p-2}} \quad \Omega;$$

$$u(x) \leq \zeta(z) \quad \text{in} \quad \Omega;$$

$$u(x) = 0 \quad \text{on} \quad \Omega.$$

Based on Ricceri's variational principle and Bonanno's three critical points theorem, we prove the existence and multiplicity of solutions under some general assumptions on the nonlinearity f, which does not satisfy the Ambrosetti-Rabinowitz condition.

Keywords: Two-Phase Problems, Recceri's variational Principle, Obstacle function

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Some Properties of Developable Surfaces

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Abstract: In this study, we reconstruct developable surfaces by the generalization of the Frenet frame and categorize their points under singularity conditions. Depending on the type of singularity, we derive a classification of developable surfaces and analyze their properties.

Keywords: Developable surface, singularity**,** Frenet frame.

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Flat Approximations of Hypersurfaces

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Abstract: In this study, we investigate the existence and uniqueness of hypersurfaces having the same normal vector. The examination is associated with the flatness of surfaces and some structures put forward by [2] for developable surfaces enabled the desired surfaces to be obtained in Euclidean space.

Keywords: Flat approximation, developable surface.

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On Selective Covering Properties in Bitopological Spaces

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Abstract: The study is a continuation of Rothberger type covering properties in bitopological spaces. We deal with (i, j)-almost Rothberger and (i, j)- weakly Rothberger spaces. We investigate the relationships between these spaces. We give some equivalences of them. We look at the behaviours of these spaces under some certain type of mappings and we take them into consideration under products and subspaces. In conclusion section, we define some weaker forms of star covering properties in bitopological spaces.

Keywords: Selection Principles, bitopological spaces, Rothberger Property

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Some Algebraic Approaches in Nearness Theory

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Abstract: This study aims to use certain group theory concepts [1] in nearness theory and move from there to some useful structures such as topological groups. In nearness theory, the concept of open set is replaced by the concept of proximity defined between the elements of the set [2,4]. From a topological viewpoint, many important concepts such as continuity are turning into new forms with this change [3]. At this time, it is notable to consider some concepts of algebra or algebraic topology to gain different ideas in nearness theory. For example, many inferences can be made about topological groups by considering the group structure in proximity spaces. It may also be interesting to observe the change in nearness theory regarding the concepts of homomorphism and isomorphism, as in the example of open set/proximity.

Keywords: Group theory, nearness theory, proximity space.

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A Numerical Approach of a Singularly Perturbed Differential-Difference Equation with Mixed Type

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Abstract: In this study, we analyze the singularly perturbed boundary value problem (SPBVP) for a semi-linear first-order differential-difference equation with mixed type delay. These problems occur in science and engineering fields such as the study of human pupil light reflex, first-exit problems in neurobiology, models of physiological processes and diseases, optimal control theory, models of climate systems, optically bistable devices and signal transmission, quantum photonic systems [1,2] and reference therein). Especially, model of nerve excitation and propagation of electronic impulses, nerves has been described these equations, which are usually known as the FitzHugh-Nagumo equations, which are considered as electric cables, through which electric current flows (for more details see [3]).

On the other hand, for small values of perturbation parameter ε, standard numerical methods for solving SPBVPs are unstable and do not give accurate results. Therefore, it is important to develop suitable numerical methods for solving these problems, whose accuracy does not depend on the parameter value ε, i.e., methods that are convergent ε - uniformly [4].

Firstly, we give some important properties of the exact solution of the problem. Next, in order to the numerical solution of this problem, we use an exponentially fitted difference scheme on a uniform mesh. This scheme is based on the finite difference method, which includes interpolation quadratic rules where the weight and the remainder term are in integral form. It has shown that it gives first-order uniform convergent with respect to the perturbation parameter in the discrete maximum norm. Furthermore, we present the numerical experiments that their results support of the theory.

Keywords: Singular perturbation, differential-difference equation, uniform convergence.

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An Exact Solutions of the Combined KdV–mKdV Equation by the Consistent Riccati Expansion Method

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Abstract: In this study, we handled the consistent Riccati expansion (CRE) method which is related to the well known Riccati equations. In this method, although we do not convert the given nonlinear partial differential equation into an ordinary differential equation with the help of wave transformations, we use the homogeneous balance method. The used method is applied to the combined KdV–mKdV equation and it is seen that the method is effective and powerful. Maple software is used to obtain the exact solutions of the equation. 3D and 2D figures of the solutions are given.

Keywords: Consistent Riccati expansion (CRE) method, nonlinear differential equations, Riccati equation, the combined KdV–mKdV equation

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Commutators Generated by Calderon–Zygmund Operators with Kernels of Dini's Type and Weighted Lipschitz Functions on Generalized Weighted Morrey Space

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Abstract: In this study, we consider the boundedness of commutators generated by Calderon–Zygmund operators with kernels of Dini's type and weighted Lipschitz functions on generalized weighted Morrey space.

Keywords: Calderon–Zygmund operators**,** weighted Lipschitz functions, generalized weighted Morrey space.

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Schatten Class Generalized Toeplitz Operators on Harmonic Bergman-Besov Spaces of the Unit Ball

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Abstract: We introduce a family of generalized positive Toeplitz operators on harmonic Bergman–Besov spaces on the unit ball of ℝ^n for all possible parameters. We give characterizations for a generalized positive Toeplitz operator on Hilbert harmonic Bergman-Besov space to be a Schatten class operator in terms of averaging functions and Berezin transforms. The membership in the Schatten classes, have been studied in various other settings; see [1], [2] and [3]. We extend those known characterizations for classical positive Toeplitz operators on harmonic Bergman spaces to the harmonic Bergman-Besov spaces.

Keywords: Toeplitz operatör, Harmonic Bergman–Besov space, Schatten class, Berezin transform

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Exact Traveling Wave Solutions for the Non-linear Couple Drinfeld-Sokolov-Wilson (DSW) Dynamical System Using Extended Jacobi Elliptic Function Expansion Method

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Abstract: The study of water waves is significant for researchers working in many branches of science. The behavior of waves can be studied by observation or experimental means, but theoretically, mathematical modeling provides solutions to many problems in physics and engineering. Progress in this field is inevitable, with those who work in mathematics, physics, and engineering putting forth interdisciplinary studies.

Jacobi elliptic functions are valuable mathematical tools that can be applied to various aspects of mathematics, physics, and ocean engineering. In this study, traveling wave solutions of the general Drinfiel'd-Sokolov-Wilson (DSW) system, introduced as a model of water waves, were obtained by using Jacobi elliptic functions and the wave dynamics were examined. The extended Jacobi elliptic function expansion method is an effective method for generating periodic solutions. It has been observed that the periodic solutions obtained by using Jacobi elliptic function expansions containing different Jacobi elliptic functions may be different and some new periodic solutions can be obtained. 3D simulations were made using Maple™ to see the behavior of the solutions obtained for different appropriate values of the parameters. 2D simulations are presented for easy observation of wave motion.

Keywords: Nonlinear evolution equation**,** Drinfiel'd-Sokolov-Wilson (DSW) system, extended Jacobi Elliptic functions, exact solution.

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Some Approximation Properties of Lupaş-Jain Operators

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Abstract: In this presentation, we investigate some approximation properties of Lupaş-Jain operators. We present the weighted uniform approximation and a preservation property of each Lupaş-Jain operator. We obtain the monotonicity of the sequence of Lupaş-Jain operators.

Keywords: Lupaş-Jain operators, monotonicity**,** modulus of continuity function, weighted uniform approximation.

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Existence Result for Elliptic Equation on Riemannian Manifolds

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Abstract: In this talk we prove the existence of non-trivial solutions to an elliptic equation on Riemannian manifolds. Our method is based on a new variational approach.

Keywords: P-laplacian operator, existence solutions, sobolev space on complete manifolds.

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Is the Shortest Path Always the Fastest?

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Abstract: In this talk, finding the shape of the curve down which a bead sliding from rest and accelerated by gravity assuming that there is no friction will slip from one point to another in the least time will be discussed. The answer to this question is the Brachistocrone curve. The term derives from the Greek "brachistos" the shortest and "chronos" time. The Brachistocrone problem was one of the earliest problems posed in calculus of variations. It is said that Galileo (1564-1642) first presented this problem.

It is also known that the cycloid is the curve which yields the quickest descent which may be handled

under the field known as the calculus of variations, or variational calculus in physics, [1]. There are several solutions to this problem, one of which is a direct application of the Euler-Lagrange differential equation. All solutions are variational in nature and have one or another connection to the calculus of variations, [2].

Keywords: The Brachistocrone curve, Euler-Lagrange equation, calculus of variations.

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Some Results on the Existence of Positive Solutions for the Iterative System of the Impulsive Boundary Value Problem

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Abstract: The third-order impulsive boundary value problems have been studied by a large number of authors. Additionally, systems of third-order impulsive boundary value problems have attracted the attention of several researchers. The importance of both theory and applications has led to interest in obtaining appropriate eigenvalue intervals for the existence of positive solutions to iterative systems with nonlinear boundary value problems. The purpose of this study is to identify the intervals of eigenvalues of the parameters $\mu_1, \mu_2, ..., \mu_n$ for which the iterative systems of the third-order impulsive boundary value problem (IBVP) have positive solutions. For this purpose, firstly we present several definitions and basic lemmas, which are important tools for our main result. And then, we construct the Green function and analyze its properties. We present the structure of the iterative system. Finally, we used various fixed-point theorems from the literature to arrive at our main result.

Keywords: Boundary value problem, fixed point theorem, impulsive differential equation, iterative system.

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An Inverse Source Problem for the Quantum Kinetic Equation

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Abstract: In this study, we deal with an inverse problem of determining source function from the quantum kinetic equation. Our purpose is to investigate the solvability conditions of the problem. We first reduce the equation to a second order partial differential equation by the Fourier transform.

Then we establish a Carleman estimate with the method given in [1]. By using this estimate, we prove the uniqueness of solution of the problem. Quantum kinetic equations are related with the ultrahyperbolic Schrödinger equations. Solvability of some inverse problems for these equations were studied by Amirov [2] and Anikonov [3]. Different types of inverse problems for hyperbolic, parabolic, elliptic type equations and non-stationary Schrödinger equation were considered in [4, 5, 6]. We refer Lavrentiev et al. [1], Amirov [2], Gölgeleyen and Yamamoto [7] for ultrahyperbolic equations and Gölgeleyen and Kaytmaz [8] for ultrahyperbolic Schrödinger equations.

Keywords: Quantum kinetik equation, inverse problem**,** Uniqueness of solution.

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An Approximate Solution for System of Delay Differential Equations with Delay Parameter

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Abstract: Delay differential equations (DDEs) are very important and useful in many areas of science and engineering, for instance, physics, biomathematics, medicine, economics, chemistry, etc [2]. For instance, process of combustion in small rockets is described by a system of delay differential equations [3]. Motivated by above works, we consider an initial value problem for system of differential equations with delay terms. In this study, the delay term r is chosen as large. Because if r is taken as small, it can be easily reduced to ordinary differential equation with the help of Taylor series, which such equations take place a lot in the literature [1]. The aim of this study is to present a more effective numerical method than classical methods. This method to be used for the numerical solution consists of a finite difference scheme given on a uniform mesh. The scheme is constructed by the method based on using appropriate quadrature rules with the weight and remainder terms in integral form. These results in a local truncation error containing only first derivative of the exact solution and hence facilitates examination of the convergence. Also we present numerical results which confirm the theoretical analysis.

Keywords: Basis functions, delay equation, finite difference**.**

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Ulam-Type Stability for Implicit Fractional Proportional Equation

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Abstract: The generalized proportional Caputo fractional derivative (GPFD) is a recently developed derivative that ex tends the classical Caputo fractional derivative, allowing for a more comprehensive representation of intricate phenomena in various fields such as physics, chemistry, and biology. This thesis first deals with the study of the existence and uniqueness of solution for the implicit in integro-differential equations involving the generalized proportional Caputo fractional derivatives. The second objective is establish the four different Ulam-type stabil ity, i.e., (Ulam-Hyers stability, generalized Ulam-Hyers stability, Ulam-Hyers-Rassias stability and generalized Ulam-Hyers-Rassias stability).

Keywords: Ulam-type stability, generalized proportional Caputo fractional derivatives, existence and uniqueness of solution, fixed point theorem.

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A New Numerical Method to Solution of Conformable Fractional Partial Differential Equation

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Abstract: In this paper, the new numerical method is used to examined the novel numerical solutions for conformable fractional partial differential equations. 2D and 3D graphs of the solutions of the equation have been drawn. The suggested method was tested using computer simulations to show that it is reliable and effective.

Keywords: Nonlinear equation**,** conformable fractional derivative, fractional differential equation.

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Investigation of the Unsteady Three-Dimensional Magnetohydrodynamic Flow Of Nanofluids Over A Decelerated Rough Rotating Disk.

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Abstract: The study deals with incompressible unsteady magnetohydrodynamic flows of Nanofluids over a decelerating rotating rough disc. The aim of this study is to investigate the effect of surface roughness on the solutions of the three-dimensional unsteady flow equations under different types of nanofluids and magnetic field intensities. Nanofluids containing copper, copper oxide and silver components are studied as a typical Von Karman flow on a rough disc. The surface roughness is mathematically modelled using the MW model and the non-slip boundary conditions are replaced by partial slip conditions. The resulting partial differential equations are transformed into ordinary differential equations using similarity transformations and solved by numerical methods. The effect of different parameters on the velocity profiles is shown graphically. The results show that the surface roughness affects the characteristics of the velocity profiles and the boundary layer thickness, and also that this effect varies significantly under different nanofluid parameters and magnetic field intensity.

Keywords: Boundary layer flow, nanoparticles, similarity solutions, decelerating rotating disk flow, magnetic field, uniform suction, unsteady magnetohydrodynamic flows.

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Investigation of Power-Law Fluids over a Decelerated Rotating Disk

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Abstract: In the paper by L.T. Watson and C. Wang [1], similarity transformations were effectively employed to analyze the nonlinear Navier-Stokes equations, which govern the behavior of Newtonian flow in decelerated rotating disks immersed in viscous fluids. Additionally, Paul's work [2] involved a comprehensive analysis of convective instability within Non-Newtonian fluids, with a specific focus on power-law fluids flowing over continuously rotating disks. This research builds upon these foundations by concentrating on the examination of Non-Newtonian flows over decelerating disks, thereby extending the scope of previous studies.

In the context of this presentation, a comprehensive review of Non-Newtonian fluids is provided, with special emphasis on power-law fluids. Basic background information on Non-Newtonian fluids is introduced, including a discussion of the power-law model for the variable viscosity parameter. Suitable similarity transformations are then introduced to convert the complex nonlinear Navier-Stokes equations governing Non-Newtonian fluids over decelerating rotating disks into a system of ordinary differential equations.

Ongoing research aims to gain in-depth insight into the dynamic behaviour of Non-Newtonian fluids over a decelerating rotating disk in the context of power-law flow models by numerically solving these complex nonlinear equations using MATLAB's bvp4c solver.

Keywords: Power-law fluid, similarity solutions**,** non-newtonian fluids.

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On New Characterizations Curves in 5- Dimensional Lorentzian Space

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Abstract: In this study, the equivalents of some characterizations given for curves in 5-dimensional Euclidean space in 5-dimensional Lorentz space are given. Additionally, some new characterizations in the 5-dimensional Lorentz space have been expressed and proven.

Keywords: Lorentz space, 5-dimensional Euclidean space, curves

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Discrete Fractional Order Economic Model in a Barabasi-Albert Graph with n=10 Nodes

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Abstract: In this study, an economic model including conformable fractional order differential equations is discussed. Applying the discretization process based on using piecewise constant arguments leads to discrete dynamical systems. The equilibrium points of the discrete system are obtained and its stability conditions are determined by using Schur-Cohn criterion. The system exhibits complex dynamical behaviour such as such as stable equilibrium point, period-2 orbit, period-4 orbit, period-8 orbit, period-16 orbit and chaos at the Nash equilibrium point with respect the change the speed of adjustment parameter v_1. Discrete Cournot duopoly game model is also considered on a Barabasi Albert graph (Scale free network) with N = 10 nodes. It is observed that the complex dynamical network exhibits similar dynamical behaviour such as stable equilibrium point, Flip bifurcation and chaos depending on the changing the coupling strength parameter cs

Keywords: Barabasi Albert graph, fractional dynamical system, economic model, stability, bifurcation

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Sound Wave Propagation Along a Coaxial Duct with a Step Discontinuity Via Wiener-Hopf Method

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Abstract: The goal of this work to reveal the effect of step discontinuity on sound propagation along a coaxial duct. For this, Wiener-Hopf method is employed to the boundary value problem which is reduced to the solution of systems of linear equations. This problem has very important applications not only in mathematics but also in engineering. It can be used to reduce the noise.

Keywords: Wiener-Hopf technique, boundary value problem, acoustic waves, step discontinuity.

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The Category of Groups $\mathbb{Z}_n$

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Abstract: In this study, we basically aim to obtain a set of (pre)algebroid samples for their use in further studies. To this end, we focus on cyclic groups $\mathbb{Z}_n$ and show that they form a category **Z** of which objects are all groups $\mathbb{Z}_n$ and morphisms are all group homomorphisms between them. In particular, we show that **Z** is a $\mathbb{Z}$-algebroid, i.e. a ringoid. Moreover, we examine and characterize homsets of **Z** and prove that each homset has a cyclic group structure. After that we obtain a countably many infinite set of subpre-$\mathbb{Z}$-algebroids, i.e. preringoids, of **Z** through narroving its homsets in a systematic way. Namely, for each positive integer t we obtain a $\mathbb{Z}$-submodule of each homset and show that the family of these submodules form a subpre-$\mathbb{Z}$-algebroid, i.e. a preringoid, $\mathbf{Z}_t$ of **Z**. Consequently, we get a family $\mathbb{Z}=\{\mathbf{Z}_t : t \in \mathbb{Z}\}$ of pre-$\mathbb{Z}$-algebroids where for different positive integers t and t' the prealgebroids $\mathbf{Z}_t$ and $\mathbf{Z}_{t'}$ are different.

Keywords: Additive category, $\mathbb{Z}$-algebroid, ringoid.

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Split (pre)Crossed Modules Over *R*-algebroids

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Abstract: In this paper, we first introduce (d-)split and (d-)split$^+$ epimorphisms in the context of algebroids and examine their categorical properties. Next, we introduce and examine (d-)split and d-split* (pre)crossed modules of *R*-algebroids. In particular, we give a necessary and sufficient condition for a morphism of pre-*R*-algebroids to be a d-split precrossed module. In addition, we examine the hierarchical relations between the categories obtained and illustrate the resultant hierarchical structure via a schematic diagram. After that, we analyse d-split and d-split* (pre)crossed modules futher and give a necessary and sufficient condition for a d-split* precrossed module to be a d-split crossed module. Moreover, we look over some results for split epimorphisms of and split (pre)crossed modules over associative *R*-algebras, as a reduced case. In particular, we see that any (d-)split epimorphism of associative *R*-algebras is (d-)split+ and thus conclude that the categories of (d-)split and (d-)split+ epimorphisms of associative *R*-algebras are the same.

Keywords: Algebroids, Crossed modules, Split epimorphisms.

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Some Inequalities for the Generalized Digamma Function

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Abstract: For $x > 0$, let $\Gamma(x)$ be the Euler's gamma function, $\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$ be the digamma function and $L(a,b) = \frac{b-a}{\log(\frac{b}{a})}$ be logarithmic mean of a and b. In this work, we have been motivated by the previous research (see [1,2]), developments in *k*-generalizations of the classical special functions such as properties on *k*-gamma function (see [3]), integral representation on *k*-digamma function $\psi_k(x)$ in [4] and inequalities obtained by Yıldırım in [5]. At first, we find some monotonicity properties of the functions related to the k-digamma function and its derivatives also known as k-polygamma functions. As immediate consequences, we get some results on these functions. Then, by using the previous results, we obtain the double-sided inequalities including the k-generalized digamma function and logarithmic means for all $k > 0$. At last, we prove the result related to logarithmic means, arithmetic means and *k*-digamma function. Also, it is worth to mention that the results in this work are k-generalizations of the classical ones.

Keywords: Complete monotonicity, mean value inequalities, k-digamma function.

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On the Rank of an Element in Free Leibniz Algebras

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Abstract: In this research, we focus on the free Leibniz algebra L_n generated by a set $X=\{x_1,...,x_n\}$ over a characteristic 0 field, denoted as K. We define an element u of L_n as homogenous if it can be expressed as a linear combination of the monomials with the same degree. The key objective is to analyze homogeneous elements within L_n of degree $m\geq 2$. A significant finding is established, demonstrating that for a homogeneous element u, any endomorphism Φ of L_n is an automorphism if and only if u is an element of $\Phi(L_n)^m$. This result offers a crucial criterion for discerning whether an endomorphism qualifies as an automorphism, hinging on the presence of the specified element within the relevant power of the image. Consequently, this outcome provides a straightforward algorithm to ascertain the rank of the homogeneous element u, defined as the minimum number of free generators on which the image of u under any arbitrary automorphism of L_n can depend. The research thus contributes valuable insights into understanding the automorphism properties and rank determination for homogeneous elements in free Leibniz algebras.

Keywords: Leibniz algebra, Fox derivative, automorphism.

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Ulam-type Stability for Implicit Two-order Fractional Integro-Differential Equation with 3-point Boundary Conditions

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Abstract: Recently, the existence of the solution and its Ulam stability for nonlinear fractional differential equation with two orders and only one point has been established. The first objective of this study is to discuss the existence and uniqueness of solutions for the implicit two-order fractional integro-differential equation with 3-point boundary conditions. The second goal is to establish the two different varieties of Ulam stability, Hyers-Ulam stability, generalized Hyers-Ulam stability.

Keywords: Fractional, differential equation, Caputo's fractional derivative, Ulam stability, two-order, fixed point theorem.

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Real Lines on Cubic Surfaces

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Abstract: The study of cubic hypersurfaces has a long history going back to Newton who studied real plane cubics in his book “Optics”. The discovery of the existence of exactly 27 complex lines on a nonsingular cubic surface given by Cayley & Salmon in 1849 can be considered as the beginning of the modern algebraic geometry. In 1863, Schlafli who first studied cubic surfaces in real setting classified nonsingular real cubic surfaces according to their number of real lines. He proved that there are five different types containing 27, 15, 7, 3 and 3 real lines. Clebsch observed that any cubic surface has a blow-up model. That is, any cubic surface is obtained by blowing up $\mathbb{P}^2$ at six points. If the six points are in general position, then the surface is nonsingular. Here "the generality” means no three of them are collinear and not all lying on a conic. Segre analyzed the five types in terms of reality of blown-up points and show that over the real numbers, the observation of Clebsch is not true anymore. That is, not all real cubic surface has a real blow-up model. In addition, he divided real lines on a cubic surface in two types: elliptic and hyperbolic. In this talk we introduce the basics of algebraic geometry and present the known facts about them in detail.

Keywords: Real Cubic Surfaces, Real Lines, Elliptic and Hyperbolic Lines

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On Investigating Numbers in the Sets of Diophantine D(∓2): New Insights

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Abstract: In the realm of number theory, Diophantine sets have long captured the imagination of mathematicians and served as a rich source of mathematical exploration. This paper embarks on an investigative journey into the enigmatic world of Diophantine D(∓2) sets' numbers, shedding new light on their intricate properties and profound connections.

The Diophantine D(∓2) sets, characterized by integer values, stand as an intriguing domain ripe for exploration. Our study delves into these sets (irrespective of their cardinality) seeking to unravel the hidden patterns and unique characteristics they hold. By scrutinizing their composition, we aim to unveil the presence of prime numbers within these sets.

Throughout our investigation, we draw upon the foundational concepts of Elementary and Algebraic Number Theory, invoking the Quadratic Reciprocity Law, Diophantine equations, and the legacy of eminent mathematicians like Gauss, Dirichlet, and Fermat. These tools and insights guide our exploration, culminating in a deeper understanding of the numbers in the set of Diophantine D(∓2) and their significance in the broader landscape of mathematics.

Keywords: Number Theory and Diophantine Equations, Modular Arithmetic and Diophantine Sets, Prime Residues in Diophantine D(m) Sets. Legendre Symbol and Quadratic Residues, Congruence Classes.

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The Role of Upper Triangular Matrices in Secure Communication with Examples

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Abstract: In the rapidly evolving landscape of digital communication and information exchange, cryptographic security stands as a cornerstone for protecting sensitive data and ensuring privacy. Cryptographic protocols, the building blocks of secure communication, rely on mathematical principles and computational techniques to thwart adversarial attempts to compromise confidentiality and integrity. One such mathematical concept that has gained prominence in cryptographic applications is the upper triangular matrix.

Upper triangular matrices, a subset of square matrices with distinctive properties, offer a unique perspective on cryptographic security. These matrices are characterized by having zero entries below the main diagonal, making them amenable to efficient computations and algorithmic transformations. As a foundation for cryptographic security, they play a multifaceted role in various aspects of secure communication.

This paper aims to elucidate the pivotal role of upper triangular matrices in cryptographic security. We explore some of their mathematical properties, including their algebraic structure and computational advantages and support them by numerical examples briefly. Moreover, we delve into their several practical applications within cryptographic protocols, highlighting how they enhance the security and efficiency of key management, encryption, and secure communication schemes.

Keywords: Upper Triangular Matrices, Cryptographic Security, Cryptographic Protocols, Key Management, Encryption, Secure Communication, Mathematical Concepts, Computational Efficiency, Privacy Protection, Digital Security.

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Apollonius Problem and Caustics of an Ellipsoid

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Abstract: In the paper we discuss Apollonius Problem on the number of normals of an ellipse passing through a given point [1]. It is known that the number is dependent on the position of the given point with respect to a certain astroida. The intersection points of the astroida and the ellipse are used to study the case when the given point is on the ellipse. The problem is then generalized for 3-dimensional space, namely for Ellipsoids. The number of concurrent normals in this case is known to be dependent on the position of the given point with respect to caustics of the ellipsoid [2,3]. If the given point is on the ellipsoid, then the number of normals is dependent on position of the point with respect to the intersections of the ellipsoid with its caustics. The main motivation of this paper is to classify all possible cases of these intersections.

Keywords: Ellipsoid, Apollonius, caustics, centro-surface, astroida, ellipse.

Acknowledgement: The study is supported by ADA University Faculty Research and Development Fund.

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The Deformation of Connections in Cocongruence $K_{(n-m)m}$

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Abstract: The Grassmann manifold $G(m,n)$ is the set of all m-dimensional planes of an n-dimensional projective space, with dim $G(m,n) = (n-m)(m+1)$.

One of the submanifolds of the Grassmann manifold is a complex of m-planes if the dimension of the complex exceeds the difference $n-m$.

We study the cocongruence of m-dimensional planes using the Cartan–Laptev method. In an n-dimensional projective space, the cocongruence of m-dimensional planes can be given by the following equations $\omega^{\alpha} = \Lambda_{\beta}^{\alpha a}\omega_{a}^{\beta}$.

Compositional equipment of a given cocongruence by fields of $(n-m-1)$-planes C_{n-m-1} ($L_m \oplus C_{n-m-1} = P_n$) and points $C = A + \lambda^a A_a$ allows one to define connections of three types in the associated bundle, and one of the three connections is average with respect to the other two. The deformation of these connections is considered and it is shown that the object of deformation is a pseudotensor.

We introduce the deformation object σ of the connection of the second type with respect to the connection of the first type. The deformation of the connection of the third type with respect to the connection of the first type is $-\sigma$, and the deformation of the connection of the third type with respect to the connection of the second type is -2σ.

Keywords: projective space, cocongruence of *m*-dimensional planes, connection, deformation of connection.

Bioconvection Near a Stretchable Boundary with Convective Boundary Conditions

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Abstract: A survey on steady magnetohydrodynamic boundary layer flow of Casson nanofluid with microorganisms near an expandable boundary in a porous medium is presented. The flow behaviour is analysed by incorporating a magnetic field applied in a direction which makes an angle $\alpha *$ with the boundary and convective conditions in temperature, nanoparticle concentration and microorganism density at boundary. The system of partial differential equations are non-dimendionalised. Non-dimensionalised nonlinear and coupled system of ordinary differntial equations are solved by exercising a numerical algorithm termed as spetral local-linearization method. The considered numerical algorithm is found to be convergent and capable of yielding accurate results. Graphical sketches of solutions obtained are produced and explained the variations in characteristics of flow, heat transfer, nanoparticle and microorganism density.

Keywords: Casson nanofluid, inclined magnetic field, linear stretching sheet, Gyrotactic microorganisms, Spectral local linearization method (SLLM).

Acknowledgement: The study is not supported by any organization.

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Positive Solutions of a Fractional Order *p*-Laplacian Boundary Value Problem

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Abstract: In this study we investigate the existence of positive solutions in a class of Caputo fractional order *p*-Laplacian boundary value problems. Using the Avery-Peterson fixed point theorem, the existence of at least three positive solutions of the model is obtained. Furthermore, we give an example to illustrate our main result.

Keywords: Boundary value problem, *p*-Laplacian, positive solutions, fixed point theorems, Caputo fractional derivative.

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Approximation by Nonlinear Szász–Mirakyan Operators of Durrmeyer Type

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Abstract: The main aim of this study is to introduce operators between Durrmeyer operators and max-product operators and to study their approximation, shape preserving, and localization properties.

Keywords: Szasz-Mirakyan Operators, Max-Product Kind, Durrmeyer operators.

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Numerical Study of a New Time-Fractional Monkeypox Model using Caputo Fractional Derivatives

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Abstract: In this study, we examine the new time-fractional model for monkeypox, utilizing Caputo fractional derivatives, specifically focusing on pregnant individuals. The proposed model incorporates the concept of optimum control inside various mathematical systems. The fractional variational iteration method (FVIM) is employed to computationally determine a numerical solution for this model. The results of the convergence investigation indicate that the obtained solution exhibits positivity and possesses a well-defined limit. Moreover, the suggested model incorporates the notion of optimal control inside different mathematical systems, such as the Fornberg-Whitham equation and the Newell-Whitehead-Segel equation. The FVIM technique is utilized to accurately estimate these variables and monitor their temporal fluctuations. The convergence of the model described in this study offers empirical support that validates the dependability and suitability of the proposed methodology. The findings show that the proposed approach is trustworthy, extremely efficient, and simple to use.

Keywords: Caputo fractional derivative, monkey-pox disease, mathematical modeling.

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Enhancing Music Streaming with Spotify's Algorithm: A Deep Dive into Recommendation Systems

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Abstract: Recommendation systems are vital software tools that help users navigate the overwhelming amount of online content by offering tailored suggestions based on their interests. These systems employ intricate algorithms and extensive datasets to discern user preferences, thereby aiding in managing information overload and bolstering platform loyalty and sales. In the realm of online music streaming, such as on Spotify, these systems play a crucial role in analyzing users' musical tastes and recommending songs that align with these tastes. This study provides a comprehensive examination of Spotify's music recommendation algorithm. By utilizing a dataset containing the audio features of thousands of songs, along with song data obtained via the Spotify API, the study focuses on delivering precise song recommendations using an item-based collaborative filtering method. This approach underscores the importance of song audio features in shaping user recommendations. Moreover, this research makes a significant contribution to the field by discussing the statistical analysis and machine learning techniques employed in developing recommendation systems. The effectiveness of these systems hinges on accurately understanding and predicting user preferences, making in-depth analysis and application in this area crucial for both academic and industrial advancement.

Keywords: Recommendation Systems, Collaborative Filtering, Machine Learning, Spotify

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A Study on Conformal Semi-Invariant Riemannian Maps to Sasakian Manifolds

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Abstract: The aim of this article is to apply the idea of "conformal semi-invariant Riemannian maps to almost Hermitian manifolds" to "almost contact metric manifolds". we present conformal semi-invariant Riemannian maps from Riemannian manifolds to Sasakian manifolds and find necessary and sufficient conditions for conformal semi-invariant Riemannian maps to be totally geodesic.

Keywords: Semi-invariant Riemannian map, Conformal semi-invariant Riemannian maps, Sasakian manifolds.

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The Exact Solutions of the Nonlinear Landau-Ginzburg-Higgs Equation

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Abstract: Theoretical, numerical and approximate solutions of nonlinear partial differential equations are very important in mathematics, engineering and physics. Applied mathematics deals with the development of exact solutions, approximate solutions or solutions methods of these equations. With the use of programmes such as Maple, Mathematica and Matlab, it has become easier to obtain exact solutions of nonlinear partial differential equations. The Landau-Ginzburg-Higgs (LGH) equation is a nonlinear partial differential equation arising in the study of soliton theory and integrable systems[1]. The LGH equation appears in nonlinear geometrical optics and superconductivity, solid state physics, quantum mechanics and fluid physics, plasma physics, chemical kinematics, chemical physics, geochemistry, geophysics, nonlinear wave equation[2]. Tanh's Method, Extended Tanh's Method and Sine-Cosine Method were utilised for the exact stable solition solutions of the LGH equation. It is obvious that the studies carried out will make a significant contribution to the fields of engineering and mathematical physics.

Keywords: The Landau-Ginzburg-Higgs (LGH) equation, Tanh's Method, Extended Tanh's Method

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On Lie Differentiability Conditions of Some Polynomial Structures in the Semi-tangent Bundle

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Abstract: In popular differential geometry, the tensor structures on smooth manifolds are remarkable geometric objects. In reality, every tensor structure is a polynomial structure. A tensor field f of type $(1,1)$ on a differentiable manifold is called a polynomial structure if it satisfies the algebraic equation $f^n + a_n f^{n-1} + ... + a_2 f + a_1 I = 0$ where I is the identity tensor of type $(1,1)$ and $a_1, a_2, ..., a_n$ are real numbers. The Lie differentiability conditions of some polynomial structures (almost contact and almost product structures) in semi-tangent bundle t(M) are examined in this study.

Keywords: Complete lift, Pull-back bundle, Polynomial structure, Semi-tangent bundle, Almost contact structure, Lie derivation, Almost product structure.

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On the Bicubic Bezier Surfaces and Paraboloids in E^3

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Abstract: In this study first we have examined the matrix representation of the bicubic Bézier surfaces in E^3. Second as the examples elliptic, hyperbolic paraboloid and Parabolic cylinder have been examined as bicubic Bézier surface. Further we give the way how to find the control points of a given elliptic, hyperbolic paraboloid and parabolic cylinder as any bicubic Bézier surface.

Keywords: Bicubic Bézier surfaces, elliptic paraboloid, hyperbolic paraboloid, parabolic cylinder.

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Some Horizontal Lift Problems in the Semi-tensor Bundle

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Abstract: We define a semi-tensor (pull-back) bundle tB of type (p,q) using the fiber bundle M over a manifold B. The results pertaining to the horizontal lifts of some tensor fields from manifold B to its semi-tensor bundle tB of type (p,q) are the focus of the current work.

Keywords: Vector field, horizontal lift, pull-back bundle, semi-tensor bundle.

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Regularized Sinc Collocation Method Applied to Fredholm Integral Equation of the First Kind

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Abstract:

Many problems in applied mathematics and engineering can be formulated as Fredholm integral equations of the first kind:

$$Kf(x) = \int_a^b k(x,t)\, f(t)dt = g(x), \qquad x \in [a,b] \tag{1}$$

where the kernel $k(.,.)$ and the right-hand side g are smooth real-valued functions.

The determination of the solution f of (1) is an ill-posed problem in the sense of Hadamard, in the sense that the solution (if it exists) does not depend continuously on the data.

In this study one of the new techniques is used to solve numerical problems involving integral equations known as regularized sinc-collocation method. This method has been shown to be a powerful numerical tool for finding accurate solutions. So, in this talk, some properties of the regularized sinc-collocation method required for our subsequent development are given and are utilized to reduce integral equation of the first kind to some algebraic equations. Then by a theorem we show error in the approximation of the solution decays at an exponential rate.

Finally, numerical examples are included to demonstrate the validity and applicability of the technique.

Keywords: Fredholm integral equation of first kind, Collocation Method, Sinc Function.

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Some Tensor Structures on Semi-tangent Bundle t(M)

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Abstract: In this paper, we study covariant derivatives of almost contact structure and almost paracontact structure with respect to ${}^{vv}X$, ${}^{cc}X$ and ${}^{HH}X$ on semi-tangent bundle.

Keywords: Tachibana operator, vishnevskii operator, almost paracontact structure, complete lift, vertical lift, projectable linear connection, semi-tangent bundle.

Acknowledgement: The study is supported by Turkish Council for Scientific and Technological Research (TBAG- 1001, MFAG-122F131).

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Diagonal Lift Problems in a Special Class of Semi-Cotangent Bundle

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Abstract: The primary goal of the current work is to investigate diagonal lift of (1,1)-tensor fields from tangent bundle T(Mn) to semi-cotangent bundle.

Keywords: Vector field, diagonal lift, horizontal lift, pull-back bundle, cross-section, semi-cotangent bundle.

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Quaternionic Normal-Direction Curves and Applications

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Abstract: The main focus of regular curve characterization in the theory of curves in Euclidean space is that it enables the classification of curves according to specific relationships. The rectifying curve, osculating curve, or normal curve, respectively, is a space curve whose position vector always lies on one of its rectifying, osculating, or normal planes. Additionally, specific curve relationships can be identified by certain connections between their Frenet vectors or curvatures. On the other hand, quaternions are widely used in mathematics and physics. Quaternions, an extension of complex numbers, are closely related to many fundamental concepts in physics. In this study, we describe a new type of associated curves in E^3 such as quaternionic normal-direction curve and quaternionic normal-donor curve. We give characterizations for these quaternionic curves. Furthermore, we obtain applications of quaternionic normal-direction curves to some special curves such as helix, slant helix, plane curve or quaternionic normal-direction (QND)-normal curves in Euclidean 3-space.

Keywords: Quaternionic curves, normal-direction curves**,** normal-donor curves, (QND)-normal curves.

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Permanents and Determinants of Some Hessenberg Matrices with Generalized Jacobsthal-Like Sequences

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Abstract: In this study, we embark on an exploration of Hessenberg matrices with a unique twist application to generalized Jacobsthal-Like sequences, a relatively uncharted territory in mathematical research. Inspired by Dr. Taşyurdu's [1] work on Hessenberg matrices and generalized Fibonacci-Like sequences, we sought to extend this approach to the realm of Jacobsthal-Like sequences.

Our investigation uncovers intriguing relationships between Hessenberg matrices and these generalized sequences, revealing an entirely distinct methodology and set of results. We define some special Hessenberg matrices and demonstrate that their determinants and permanents are interestingly tied to the terms of generalized Jacobsthal-Like sequences, a revelation that diverges from previous findings in the field.

Definition 1 [2] Generalized Jacobsthal-Like sequence $\{K_n\}$ associated with Jacobsthal and Jacobsthal Lucas sequences is defined by recurrence relation.

$$K_n = K_{n-1} + 2K_{n-2}, \qquad n \geq 2 \tag{1.1}$$

with initial conditions $K_0 = 2s$ *and* $K_1 = s+1$, *s* being a fixed positive integer.

Definition 2 [3] Generalized Jacobsthal-Like sequence $\{V_n\}$ associated with Jacobsthal and Jacobsthal-Lucas sequences is defined by recurrence relation.

$$V_n = V_{n-1} + 2V_{n-2}, \qquad n \geq 2 \tag{1.2}$$

with initial conditions $V_0 = 2$ and $V_1 = m+1$, *m* being a fixed positive integer.

The relation between Jacobsthal and Jacobsthal-Lucas sequences and generalized Jacobsthal-Like sequences can be written as

$$K_n = J_n + sj_n,$$

$$V_n = mJ_n + j_n$$

respectively, where s, m being fixed positive integers. A few terms of these sequences are.

$$\{K_n\} = \{2s, 1+s, 1+5s, 3+7s, 5+17s, \dots\}$$

$$\{V_n\} = \{2, 1+m, 5+m, 7+3m, 17+5m, \dots\}$$

We define two types lower Hessenberg matrices and show that determinants of these Hessenberg matrices give terms of generalized Jacobsthal-Like sequences $\{K_n\}$ and $\{V_n\}$.

Keywords: Generalized Jacobsthal-Like sequence, Generalized Jacobsthal-Lucas Like sequences, Hessenberg matrices.

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Covariant Derivatives of Structures on Semi-tangent Bundle

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Abstract: We examine covariant derivates of geometric structures with regard to the horizontal, complete and vertical lift of (1, 0)−tensor field X for semi-tangent bundle in this study.

Keywords: Covarient Derivative, Almost paracontact structure, Horizontal lift, Almost contact structure, Semi-tangent bundle, Projectable linear connection.

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Short Notes for Tangent Indicatrix and Binormal Indicatrix

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Abstract: In this study, generalized Fermi-Walker derivative, generalized Fermi-Walker parallelism, and generalized nonrotating frame are investigated along the tangent indicatrix and binormal indicatrix of any curve in Euclidean 3-space. Initially, we studied the conditions of the generalized Fermi-Walker parallelism of any vector field along the tangent indicatrix in Euclidean space by considering the Frenet frame. We investigated whether the Frenet frame is a generalized non-rotating frame along the tangent indicatrix with the choice of tensor field or not. Similar investigations were made along binormal indicatrix.

Keywords: Generalized Fermi-Walker derivative, generalized non-rotating frame, tensor field, Frenet frame, tangent indicatrix, binormal indicatrix.

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Pseudo-Riemannian Metric and Lorentzian Almost r-Paracontact Structure on the Semi-tangent Bundle

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Abstract: We offer the Lorentzian almost r – paracontact structure of semi-tangent bundle tB by using the pseudo-Riemannian metric. As a beginning, we identify the pseudo-Riemannian metric of semi-tangent bundle tB with respect to the almost r – paracontact structure. Secondly we find out conditions on the Lorentzian almost r – paracontact structure when g is pseudo-Riemannian metric. Besides, We acquire some results related to Lorentzian almost r – paracontact structure and a new example for pseudo-Riemannian metric in this study.

Keywords: Almost r – paracontact structure, pseudo-Riemannian metric, pull-back bundle, semi-tangent bundle.

Acknowledgement: The study is supported by the Scientific and Technological Research Council of Türkiye. (TUBİTAK) (Project number: MFAG-122F131).

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Short Time Behaviour of a Three-dimensional Dam-break Flow Involving Two Fluids with Free Surfaces

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Abstract: In this work, we consider the short time behaviour of the three-dimensional flow of two immiscible fluids with finite depth within the framework of potential theory. The gravity-driven potential flow that results from the disappearance of a vertical cylindrical dam of circular cross-sections surrounded by fluids of different densities is studied. Specifically, we are interested in the case when liquids have the same depth and lie inside and outside the cylindrical dam. Attention is paid to the motion of free surfaces, the interface between fluids, and the singular behaviour of the velocity field at their intersections.

To begin, we derive the leading-order solution by employing asymptotic analysis. This approach utilizes a small parameter to account for the short duration of this stage. The linear problem is solved by the Fourier series method. In the limiting case, where the radius and centre of the cylinder goes to infinity, the problem is reduced to the initial stages of the two-dimensional dam-break problem (see [6] for more details). Finally, we will present both analytical and computational results.

Keywords: Potential theory, gravity-driven flow, asymptotic analysis.

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Hyperbolic Pell Polynomials and Their Some Properties

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Abstract: This study proposes introduce of the Hyperbolic Pell polynomials. In this study, we give the generating functions and the Binet formulas of Hyperbolic Pell polynomials. We also obtain some important identities involving the Hyperbolic Pell polynomials. In future we shall further develop some results concerning the terms of these polynomials.

Keywords: Pell number, Pell polynomials, Hyperbolic Pell number, Hyperbolic Pell polynomials.

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Characterization of Special Space Curves with Euler Angles

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Abstract: Curvature and torsion, in their most general forms, are defined using skew-symmetric Cartan matrix, which leads to the Serret Frenet equations. Euler angles are used to develop simple and general expressions for elements of a curve with respect to Cartan matrix. In this study, special space curves (general helix, slant helix, glad helix et.) be characterize with the aid of curvature and torsion equations, which are the results of the Serret-Frenet equations obtained by using the Cartan matrix created with Euler angles.

Keywords: Euler angle, Cartan matrix, general helix, slant helix**.**

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Acceleration Slant Curve in E_1^3

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Abstract: In this study, we introduce define an acceleration slant curve, where the scalar product of its acceleration vector and a unit fixed direction remains constant in Minkowski 3-space. Additionally, we provide a characterization based on the curvatures of this acceleration slant curve. After that, we examine connections between the acceleration slant curve and several distinctive curve types, including spherical curves, helices, slant helices, and Salkowski curves. To further illustrate these concepts, we used Mathematica.

Keywords: Acceleration, helix, slant helix, Minkowski 3-space.

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A Modelling of the Natural Logarithm and Mercator Series as 5th , 6th, and 7th Order Bézier Curve in Plane

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Abstract: In this study, natural logarithm function $f(x) = lnx$ with base $\vec{e}$ has been examined as polynomial function and 5th , 6th and 7th order Bézier curve. By modelling matrix representation of 5th , 6th and 7th order Bézier curve we have found the control points in plane. Further, Mercator series for $f(x) = ln(1 + x)$, and $f(x) = ln(1 - x)$ have been written as the polynomial functions as 5th , 6th and 7th order Bézier curve in plane based on the control points with matrix form in E^2. At least the control points of $f(x) = ln(1 - x^2)$ curve as the polynomial functions as 5th , 6th and 7th order Bézier curve in plane are examined based on the control points with matrix form in E^2. We have found 6th order Bézier curve in plane.

Keywords: Natural logarithm curve, Bézier curves, Mercator series.

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Quaternions and Three-Dimensional Rotation

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Abstract: In this study, the basic definitions and concepts related to quaternions are explained and it is stated that each unit quaternion corresponds to rotations in three-dimensional Euclidean space. $\mathbb{H}_1$ denotes the set of unit quaternions, it has been stated that for the unit quaternion $q = cos\theta + nsin\theta \in \mathbb{H}_1$, the rotation transformation $R_q(u) = quq^{-1}$, $u \in \mathbb{R}^3$ refers to the rotation around the rotation axis $n \in \mathbb{R}^3$ up to angle 2θ.

Keywords: Quaternions, spherical geometry, rotation transformation.

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Approximate Analytical and Numerical Solutions of the Fractional Black-Scholes Option Pricing Equation

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Abstract: The investigation of numerical solutions of equations that cannot be solved analytically has contributed greatly to the development of mathematical modeling theory. Analytical and approximate solution methods can be recommended to solve fractional partial differential equations, fractional differential equations, and integral equations. The Black-Scholes equation, which is widely used in option pricing models, is considered and a transition is made from this equation to the fractional Black-Scholes equation. Options, on the other hand, are an agreement between two parties that gives the buyer the right to buy or sell any product in the future at a price determined today (strike price).

Solutions of the fractional Black-Scholes option pricing model are obtained by using the Adomian Decomposition Method (ADM), Variational Iteration Method (VIM), Laplace Homotopy Perturbation Method (LHPM) and Finite Difference Approach (FDA), which are approximate analytical and numerical solution methods of fractional partial differential equations. The solution functions obtained using these methods are interpreted with graphics and fractional derivatives are expressed based on Caputo fractional derivative.

Keywords: Fractional partial differential equation, Black-Scholes option pricing model, fractional Black-Scholes equation, Caputo fractional derivative.

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Fundamental Equations on Conformal Fedosovov Spaces

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Abstract: The present work is devoted to the study of the basic equations of conformally Fedosov structures [1]. These equations are obtained in the form of closed linear equations in covariant derivatives of the Cauchy type. It is established that the general solution depends on no more than $\frac{1}{2}n(n+1)$ numerical parameters. The maximum is achieved in projectively Euclidean spaces. Estimates are found for these equations dependence on spaces that are not projectively Euclidean.

We obtain the following results.

Theorem. In the n-dimensional non-projective euclidean space, $n \geq 4$, the general solution of the fundamental equation depended on independent parameters $r \leq \frac{1}{2} n(n-3) + 2$.

Keywords: Conformally Fedosov structures, general solution, independence parameters.

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Data-Oriented Bayesian Network Model of Breast Cancer Risks

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Abstract: The World Health Organization (WHO) reports that breast cancer is the most common type of cancer in women. The risks that trigger breast cancer are analyzed using the data set published by the American Breast Cancer Surveillance Consortium (BCSC). Bayesian Network (BN), a probabilistic graph model that supports the inferential processes of applied graphical data science, is used. The relationship between the risk factors identified by BCSC (age groups, density, race, BMI, childbearing status, cancer history, surgical menopause, hormone therapy) and breast cancer is modeled with BNs. Structure learning algorithms are used to model data-driven breast cancer risks with probabilistic information using BNs. Structure learning algorithms are used to model data-driven breast cancer risks with probabilistic information using BNs. The dataset of values from 18149 individual examinations has been made available for open access by BCSC. The detection of breast cancer was confirmed by pathology and mammography images. The best prediction among the algorithms for learning structure from data is given by the Greedy Thick Thinning (GTT) algorithm.

Keywords: Bayesian network, breast cancer, inferences, probabilistic graphical models.

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Some Indexes for Ditopological Texture Graphs

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Abstract: In this study, some ındexes of ditopological texture graphs are obtained. In addition to, some properties of these ındexes for ditopological texture graphs are investigated.

Keywords: Texture graph, ditopology, index.

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FULL TEXTS

Shehu Variational Iteration Method for Solutions of Schrödinger Equations

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Abstract: In this study, approximate and exact solutions of three dimensional time dependent Schrödinger equations have examined using the Shehu variational iteration method. Different iteration formulas have been formed for the Shehu variational iteration method. The accuracy of the approximate and exact solutions of the Shehu variational iteration method is given with numerical examples. Approximate solutions of example problems of three-dimensional time dependent Schrödinger equations have formed by using different iteration formulas of the Shehu variational iteration method. It has been shown that approximations that converge to the exact solution more rapid can be obtained depending on the selection of iteration formula used. With the graphics plots, the imaginary and real parts of the exact solution and approximate solutions have compared and absolute error graphs are given. With the compatibility of the results, it has been observed that the Shehu variational iteration method provides rapid approximations to the exact solution.

Keywords: Schrödinger equation, Shehu variational iteration method, Lagrange multiplier.

1 Introduction

Most problems in science and engineering are expressed by linear or nonlinear ordinary or partial differential equations. In order to provide more important information about scientific questions, it is necessary to solve these problems correctly. For this purpose, various researches have been done, and many analytical and numerical methods have been developed using integral transformations. Such as: Sumudu transform, Elzaki transform, Natural transform, G-transform, Sawi transform, Shehu transform, and others [1-3].

The Laplace type integral transform called the Shehu transform, which is a generalization of the Laplace and Sumudu integral transforms, was introduced by Maitama in 2019 [4]. Later, Chu and et.al [5] used the Shehu decomposition and variational iteration transform method to solve fractional differential equations. Kapoor and et.al [6] applied Shehu transform to fractional hyperbolic telegraph equations. Boulekhras and Belghaba used Shehu variational iteration method to solve fractional differential equations [7].

In this study, we will investigate three dimensional linear and nonlinear time dependent Schrödinger equations given as follows

$$\left(i\frac{\partial}{\partial t}+\nabla^2-V(x,y,z)\right)\psi(x,y,z,t)=0 \quad (1)$$

$$\left(i\frac{\partial}{\partial t}+\nabla^2-V(x,y,z)-\beta|\psi(x,y,z,t)|^2\right)\psi(x,y,z,t)=0 \quad (2)$$

where $2m=\hbar=1$ [8], $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$ is the Laplace operatör, $V(x,y,z)$ is the potential, $\psi(x,y,z,t)$ is solutions function and β is real constant. The Shehu variational iteration method was applied to the three-dimensional Schrödinger equations. Some preliminary informations, numerical examples and graphics are included to demonstrate the accuracy of the method.

2 Preliminaries

Definition 2.1 [4] The Shehu transform of the function $f(t)$ is defined over the set of functions

$$A=\left\{f(t)|\ \exists M,\tau_1,\tau_2>0,|f(t)|<Me^{|t|/\tau_j},t\in(-1)^j\times[0,\infty)\right\} \quad (3)$$

by the following

$$S[f(t)]=F(s,u)=\int_0^\infty e^{-\frac{st}{u}}f(t)\,dt\ ,\ t\geq 0,\ s\geq 0. \quad (4)$$

Definition 2.2 [4] The inverse Shehu transform is given by

$$S[f(t)]=F(s,u) \quad (5)$$

and

$$S^{-1}[F(s,u)]=f(t)\ ,\ t\geq 0. \quad (6)$$

Definition 2.3 [4] Let functions $f(t)$ and $g(t)$ be in set A. Hence, linearity property of Shehu transform is

$$S[\alpha f(t)+\beta g(t)]=\alpha S[f(t)]+\beta S[g(t)] \quad (7)$$

where α and β are nonzero arbitrary constants.

Definition 2.4 [4] If function $f^n(t)$ is the nth derivative of the function $f(t)\in A$ with respect to $'t'$, then Shehu transform of $f^n(t)$ is defined by

$$S[\,f^n(t)]=\frac{s^n}{u^n}F(s,u)-\sum_{k=0}^{n-1}\left(\frac{s}{u}\right)^{n-(k+1)}f^k(0). \quad (8)$$

When $n=1,2$ and 3 in equation (8) above, derivatives with respect to t are as follows

$$S[\,f'(t)]=\frac{s}{u}F(s,u)-f(0) \quad (9)$$

$$S[\,f''(t)]=\frac{s^2}{u^2}F(s,u)-\frac{s}{u}f(0)-f'(0) \quad (10)$$

$$S[\,f'''(t)]=\frac{s^3}{u^3}F(s,u)-\frac{s^2}{u^2}f(0)-\frac{s}{u}f'(0)-f''(0). \quad (11)$$

Definition 2.5 [4] f and g be defined in A having Shehu transform $A(s,u)$ ve $B(s,u)$. Then a Shehu transform of the convulation of f and g

$$(f * g)(t) = \int_0^t f(x)g(t-x)\,dx \tag{12}$$

is given by

$$S[(f * g)(t)] = A(s,u)B(s,u). \tag{13}$$

3 Shehu Variational Iteration Method

To illustrate the basic idea of this method, we consider a general nonlinear differential equation as follow

$$(L+N)u(t) = f(t) \tag{14}$$

$f(t)$ is analytical function. L and N are linear and nonlinear differential operator. The correction functional for (14) is

$$u_{n+1}(t) = u_n(t) + \int_0^t \lambda(\varepsilon)\big(Lu_n(\varepsilon) + N\widetilde{u_n}(\varepsilon) - f(\varepsilon)\big)d\varepsilon, n \geq 0. \tag{15}$$

λ the general Lagrange multiplier, defined by variational theory, $\widetilde{u_n}$ restricted variation, i.e. $\delta\widetilde{u_n} = 0$. We assume the Lagrange multiplier expressed in form

$$\lambda = \lambda(t-\varepsilon) \tag{16}$$

$n \geq 1$, u_n is the nth approximations. Applying the Shehu transform on both sides of (15), the correction functional is

$$S[u_{n+1}(t)] = S[u_n(t)] + S\left[\int_0^t \lambda(\varepsilon)\big(Lu_n(\varepsilon) + N\widetilde{u_n}(\varepsilon) - f(\varepsilon)\big)d\varepsilon\right]. \tag{17}$$

Thus,

$$\begin{aligned} S[u_{n+1}(t)] &= S[u_n(t)] + S[\lambda(t)] * S[Lu_n(t) + N\widetilde{u_n}(t) - f(t)] \\ &= S[u_n(t)] + S[\lambda(t)]S[Lu_n(t) + N\widetilde{u_n}(t) - f(t)]. \end{aligned} \tag{18}$$

Take variation with respect to $u_n(t)$ to find $\lambda = \lambda(t-\varepsilon)$

$$\frac{\delta}{\delta u_n}S[u_{n+1}(t)] = \frac{\delta}{\delta u_n}S[u_n(t)] + \frac{\delta}{\delta u_n}S[\lambda(t)]S[Lu_n(t) + N\widetilde{u_n}(t) - f(t)]$$

$$S[\delta u_{n+1}(t)] = S[\delta u_n(t)] + S[\lambda(t)]S[\delta u_n(t)]. \tag{19}$$

Applying the inverse Shehu transform to both sides of the equation gives the value of λ. Substituting the values of λ into functional (17), we get the iteration formula. From here, a first approximation function u_0 is selected to satisfy the initial or boundary conditions. With the u_0 approximation and the iteration formula, the successive approximations of u_{n+1} are easily found. The solution is

$$u(t) = \lim_{n\to\infty} u_n(t). \tag{20}$$

4 Application of the method to the linear Schrödinger equation

We consider following 3D linear Schrödinger equation

$$i\frac{\partial\psi(x,y,z,t)}{\partial t}+\left(\nabla^2-V(x,y,z)\right)\psi(x,y,z,t)=0 \tag{21}$$

The correction functionals for equation (21) are as follows

$$\psi_{n+1}(x,y,z,t)=\psi_n(x,y,z,t)+\int_0^t\lambda_1(t-\varepsilon)\left(i\frac{\partial\psi_n(x,y,z,\varepsilon)}{\partial\varepsilon}+\left(\nabla^2-V(x,y,z)\right)\widetilde{\psi_n}(x,y,z,\varepsilon)\right)d\varepsilon\,, \tag{22}$$

$$\psi_{n+1}(x,y,z,t)=\psi_n(x,y,z,t)+\int_0^x\lambda_2(x-\zeta)\left(\frac{\partial^2\psi_n(\zeta,y,z,t)}{\partial\zeta^2}+\left(i\frac{\partial}{\partial t}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-V(\zeta,y,z)\right)\widetilde{\psi_n}(\zeta,y,z,t)\right)d\zeta,\ n\geq 0. \tag{23}$$

Applying the Shehu variational iteration method to both sides of equations (22) and (23), we get the correction functionals in the following

$$S[\psi_{n+1}(x,y,z,t)]=S[\psi_n(x,y,z,t)]+S\left[\int_0^t\lambda_1(t-\varepsilon)\left(i\frac{\partial\psi_n(x,y,z,\varepsilon)}{\partial\varepsilon}+\left(\nabla^2-V(x,y,z)\right)\widetilde{\psi_n}(x,y,z,\varepsilon)\right)d\varepsilon\right] \tag{24}$$

$$S[\psi_{n+1}(x,y,z,t)]=S[\psi_n(x,y,z,t)]+S\left[\int_0^x\lambda_2(x-\zeta)\left(\frac{\partial^2\psi_n(\zeta,y,z,t)}{\partial\zeta^2}+\left(i\frac{\partial}{\partial t}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-V(\zeta,y,z)\right)\widetilde{\psi_n}(\zeta,y,z,t)\right)d\zeta\right],\ n\geq 0. \tag{25}$$

Using the properties of the Shehu transformation we get the following,

$$S[\psi_{n+1}(x,y,z,t)]=S[\psi_n(x,y,z,t)]+S\left[\lambda_1(t)*\left(i\frac{\partial\psi_n(x,y,z,t)}{\partial t}+\left(\nabla^2-V(x,y,z)\right)\widetilde{\psi_n}(x,y,z,t)\right)\right]$$

$$=S[\psi_n(x,y,z,t)]+S[\lambda_1(t)]S\left[i\frac{\partial\psi_n(x,y,z,t)}{\partial t}+\left(\nabla^2-V(x,y,z)\right)\widetilde{\psi_n}(x,y,z,t)\right]$$

$$=S[\psi_n(x,y,z,t)]+S[\lambda_1(t)]\left(i\left(\frac{s}{u}S[\psi_n(x,y,z,t)]-\psi_n(x,y,z,0)\right)+S\left[\left(\nabla^2-V(x,y,z)\right)\widetilde{\psi_n}(x,y,z,t)\right]\right), \tag{26}$$

$$S[\psi_{n+1}(x,y,z,t)]=S[\psi_n(x,y,z,t)]+S\left[\lambda_2(x)*\left(\frac{\partial^2\psi_n(x,y,z,t)}{\partial x^2}+\left(i\frac{\partial}{\partial t}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-V(x,y,z)\right)\widetilde{\psi_n}(x,y,z,t)\right)\right]$$

$$= S[\psi_n(x,y,z,t)] + S[\lambda_2(x)]S\left[\frac{\partial^2\psi_n(x,y,z,t)}{\partial x^2}\right.$$

$$\left. + \left(i\frac{\partial}{\partial t} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - V(x,y,z)\right)\widetilde{\psi_n}(x,y,z,t)\right)\Bigg]$$

$$= S[\psi_n(x,y,z,t)] + S[\lambda_2(x)]\left(\frac{s^2}{u^2}S[\psi_n(x,y,z,t)] - \frac{s}{u}\psi_n(0,y,z,t)\right.$$

$$\left. - \frac{\partial\psi_n(x,y,z,t)}{\partial x}\Big|_{x=0} + S\left[\left(i\frac{\partial}{\partial t} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - V(x,y,z)\right)\widetilde{\psi_n}(x,y,z,t)\right]\right), \quad (27)$$

We take variation with respect to ψ_n of both sides of (26)-(27) and with known $\delta\psi_n(x,y,z,t) = 0$,

$$S[\delta\psi_{n+1}(x,y,z,t)] = S[\delta\psi_n(x,y,z,t)] + \delta\left(S[\lambda_1(t)]i\left(\frac{s}{u}S[\psi_n(x,y,z,t)] - \psi_n(x,y,z,0)\right)\right)$$

$$= \left(1 + i\frac{s}{u}S[\lambda_1(t)]\right)S[\delta\psi_n(x,y,z,t)] \quad (28)$$

$$S[\delta\psi_{n+1}(x,y,z,t)] = S[\delta\psi_n(x,y,z,t)] + \delta\left(S[\lambda_2(x)]\left(\frac{s^2}{u^2}S[\psi_n(x,y,z,t)] - \frac{s}{u}\psi_n(0,y,z,t) - \frac{\partial\psi_n(x,y,z,t)}{\partial x}\Big|_{x=0}\right)\right)$$

$$= \left(1 + \frac{s^2}{u^2}S[\lambda_2(x)]\right)S[\delta\psi_n(x,y,z,t)]\,. \quad (29)$$

$n \geq 0$. Therefore, we have stationary conditions

$$S[\lambda_1(t)] = i\frac{u}{s}\ ,\ S[\lambda_2(x)] = -\frac{u^2}{s^2}\,. \quad (30)$$

Taking the Shehu inverse from both sides of the (30) equation

$$\lambda_1(t) = i\ ,\ \lambda_2(x) = -x. \quad (31)$$

Substituting λ values into correction functionals, results become as follow

$$S[\psi_{n+1}(x,y,z,t)] = S[\psi_n(x,y,z,t)] + S\left[\int_0^t i\left(i\frac{\partial\psi_n(x,y,z,\varepsilon)}{\partial\varepsilon} + \left(\nabla^2 - V(x,y,z)\right)\psi_n(x,y,z,\varepsilon)\right)d\varepsilon\right] \quad (32)$$

$$S[\psi_{n+1}(x,y,z,t)] = S[\psi_n(x,y,z,t)] + S\left[\int_0^x (\zeta - x)\left(\frac{\partial^2\psi_n(\zeta,y,z,t)}{\partial\zeta^2}\right.\right.$$

$$\left.\left. + \left(i\frac{\partial}{\partial t} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - V(\zeta,y,z)\right)\psi_n(\zeta,y,z,t)\right)d\zeta\right],\ n \geq 0. \quad (33)$$

We applying the inverse Shehu transform on both sides of equation (32) and (33),

$\psi_{n+1}(x,y,z,t) = \psi_n(x,y,z,t)$

$$+S^{-1}\left[i\frac{u}{s}S\left[i\frac{\partial\psi_n(x,y,z,t)}{\partial t}+\left(\nabla^2-V(x,y,z)\right)\psi_n(x,y,z,t)\right]\right], \quad (34)$$

$$\psi_{n+1}(x,y,z,t)=\psi_n(x,y,z,t)+S^{-1}\left[-\frac{u^2}{s^2}S\left[\frac{\partial^2\psi_n(x,y,z,t)}{\partial x^2}\right.\right.$$

$$\left.\left.+\left(i\frac{\partial}{\partial t}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-V(x,y,z)\right)\psi_n(x,y,z,t)\right]\right], n\geq 0. \quad (35)$$

Iteration formulas are as follows

$$\psi_{n+1}(x,y,z,t)=\psi_n(x,y,z,0)+S^{-1}\left[i\frac{u}{s}S\left[\left(\nabla^2-V(x,y,z)\right)\psi_n(x,y,z,t)\right]\right], \quad (36)$$

$$\psi_{n+1}(x,y,z,t)=\psi_n(0,y,z,t)+x\frac{\partial\psi_n(x,y,z,t)}{\partial x}\Big|_{x=0}$$

$$+S^{-1}\left[-\frac{s^2}{u^2}S\left[\left(i\frac{\partial}{\partial t}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-V(x,y,z)\right)\psi_n(x,y,z,t)\right]\right], n\geq 0. \quad (37)$$

5 Application of the method to the nonlinear Schrödinger equation

With $\lambda_1(t)=i$ and $\lambda_2(x)=-x$, we obtain the following iteration formula for the nonlinear Schrödinger equation

$$\psi_{n+1}(x,y,z,t)=\psi_n(x,y,z,0)+S^{-1}\left[i\frac{u}{s}S[(\nabla^2-V(x,y,z)-\right.$$

$$\left.\beta|\psi_n(x,y,z,t)|^2)\psi_n(x,y,z,t)]\right], \quad (38)$$

$$\psi_{n+1}(x,y,z,t)=\psi_n(0,y,z,t)+x\frac{\partial\psi_n(x,y,z,t)}{\partial x}\Big|_{x=0}+S^{-1}\left[-\frac{s^2}{u^2}S\left[\left(i\frac{\partial}{\partial t}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right.\right.\right.$$

$$\left.\left.\left.-V(x,y,z)-\beta|\psi_n(x,y,z,t)|^2\right)\psi_n(x,y,z,t)\right]\right], n\geq 0. \quad (39)$$

Example 1. Consider the following problem for 3D linear Schrödinger equation

$$\left(i\frac{\partial}{\partial t}+\nabla^2\right)\psi(x,y,z,t)=0\ ,\ t>0\ ,\ x,\ y,\ z\in[0,2\pi) \quad (40)$$

$$\psi(x,y,z,0)=\sin x\sin y\sin z\ , \quad (41)$$

$$\psi(0,y,z,t)=0\ ,\ \psi_x(0,y,z,t)=e^{-3it}\sin y\sin z\ ,$$

$$\psi(x,0,z,t)=0\ ,\ \psi_y(x,0,z,t)=e^{-3it}\sin x\sin z\ , \quad (42)$$

$$\psi(x,y,0,t)=0\ ,\ \psi_z(x,y,0,t)=e^{-3it}\sin x\sin y\ ,$$

potential

$$V(x,y,z)=0 \quad (43)$$

Using the iteration formula

$$\psi_{n+1}(x,y,z,t)=\sin x\sin y\sin z+S^{-1}\left[i\frac{u}{s}S[\nabla^2\psi_n(x,y,z,t)]\right] \quad (44)$$

and

$\psi_0(x,y,z,t) = \sin x \sin y \sin z$,

the approximations obtained as follows

$$\psi_1(x,y,z,t) = (1-3it)\psi_0(x,y,z,t)$$

$$\psi_2(x,y,z,t) = \left(1-3it-\frac{9t^2}{2!}\right)\psi_0(x,y,z,t)$$

$$\vdots$$

$$\psi_n(x,y,z,t) = \left(1-3it-\frac{9t^2}{2!}+\cdots+\frac{(-3it)^n}{n!}\right)\psi_0(x,y,z,t) \quad (45)$$

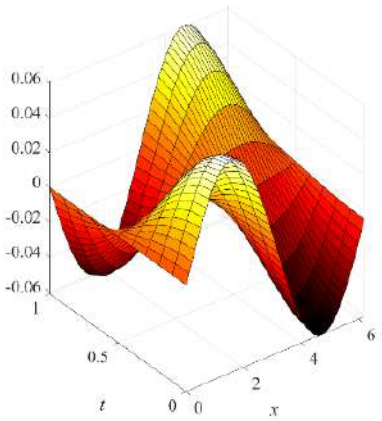
The solution is

$$\psi(x,y,z,t) = \lim_{n\to\infty} \psi_n(x,y,z,t) = \psi_e(x,y,z,t)\ . \quad (46)$$

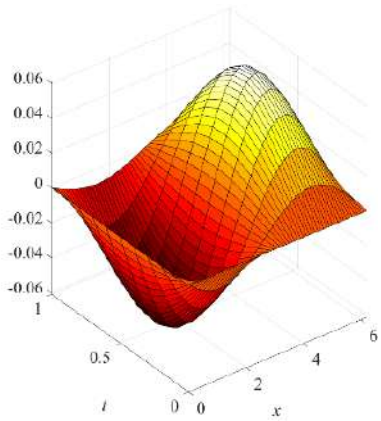
Here

$$\psi_e(x,y,z,t) = e^{-3it}\sin x \sin y \sin z \quad (47)$$

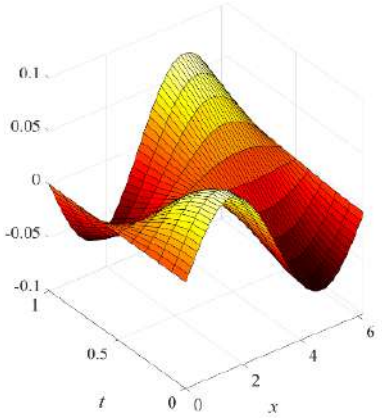
is the exact solution.



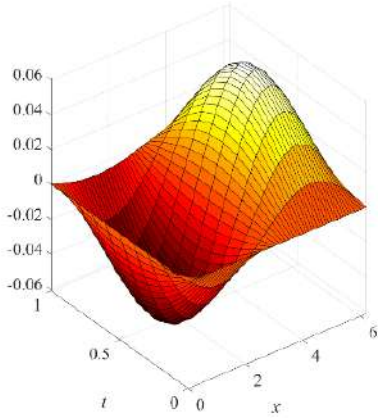
(a) $\mathrm{Re}[\psi_e]$



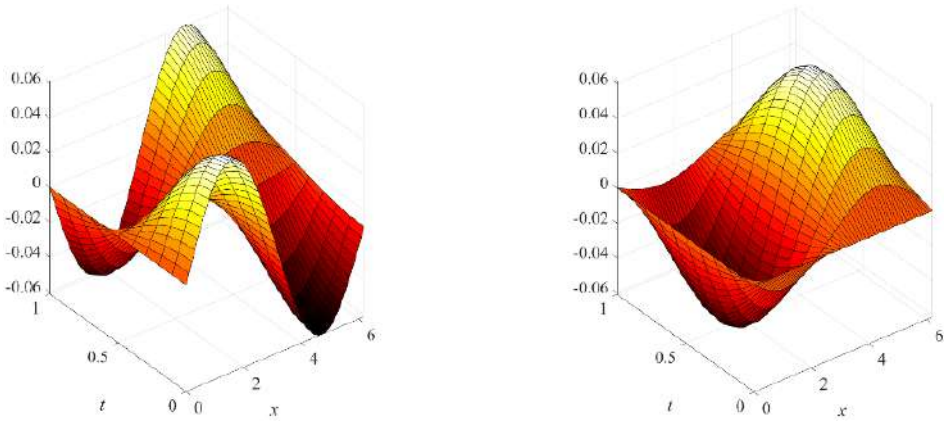
(b) $\mathrm{Im}[\psi_e]$



(c) $\mathrm{Re}[\psi_{7(45)}]$



(d) $\mathrm{Im}[\psi_{7(45)}]$



(e) $\mathrm{Re}[\psi_{7(49)}]$ **(f)** $\mathrm{Im}[\psi_{7(49)}]$

Figure 1. The graphs of the real and imaginary parts of exact solution u_e in (47) and approximate solutions $u_{7(41)}$, $u_{7(46)}$ in (45), (49) for $n = 7$ at $y, z = 0.25$ in Example 1.

If we use the iteration formula

$$\psi_{n+1}(x,y,z,t) = xe^{-3it}\sin y \sin z + S^{-1}\left[-\frac{s^2}{u^2}S\left[\left(i\frac{\partial}{\partial t}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\psi_n(x,y,z,t)\right]\right], n \geq 0, \quad (48)$$

and

$\psi_0(x,y,z,t) = xe^{-3it}\sin y \sin z$

the approximations are obtained as follows

$\psi_1(x,y,z,t) = \left(x - \frac{x^3}{3!}\right)e^{-3it}\sin y \sin z$

$\psi_2(x,y,z,t) = \left(x - \frac{x^3}{3!} + \frac{x^5}{5!}\right)e^{-3it}\sin y \sin z$

$\vdots$

$$\psi_n(x,y,z,t) = \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + (-1)^n\frac{x^{2n+1}}{(2n+1)!}\right)e^{-3it}\sin y \sin z \quad (49)$$

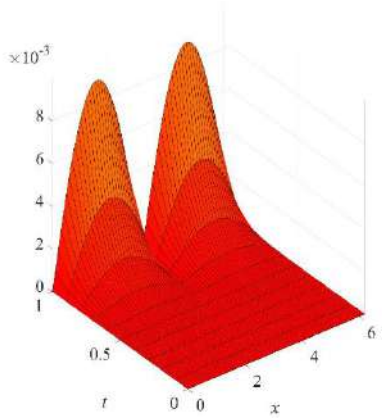
We get the following solution

$\psi(x,y,z,t) = e^{-3it}\sin x \sin y \sin z.$

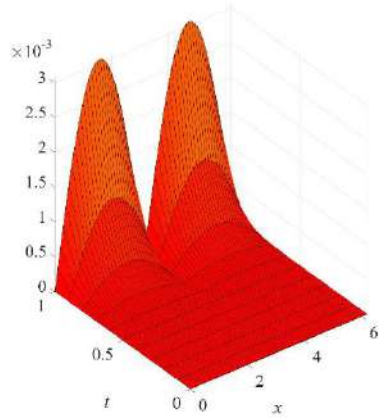
t	x	Exact solution $\mathrm{Re}[\psi_e]$	SVIM solution $\mathrm{Re}[\psi_{7(45)}]$	SVIM solution $\mathrm{Re}[\psi_{7(49)}]$	Absolute error $\lvert\mathrm{Re}[\psi_e - \psi_{7(45)}]\rvert$	Absolute error $\lvert\mathrm{Re}[\psi_e - \psi_{7(49)}]\rvert$
0.1	0.5	2.8034×10^{-2}	2.8034×10^{-2}	2.8034×10^{-2}	0	0
0.2	1.0	4.2509×10^{-2}	4.2509×10^{-2}	4.2509×10^{-2}	0	0

0.3	1.5	3.7953×10^{-2}	3.7952×10^{-2}	3.7953×10^{-2}	6.4601×10^{-7}	0
0.4	2.0	2.0168×10^{-2}	2.0162×10^{-2}	2.0168×10^{-2}	5.8414×10^{-6}	0
0.5	2.5	2.5912×10^{-3}	2.5685×10^{-3}	2.5912×10^{-3}	2.2712×10^{-5}	0
0.6	3.0	-1.9625×10^{-3}	-1.9853×10^{-3}	-1.9625×10^{-3}	2.2779×10^{-5}	0
0.7	4.0	2.3386×10^{-2}	2.3800×10^{-2}	2.3387×10^{-2}	4.1394×10^{-4}	1.4253×10^{-6}
0.8	4.5	4.4121×10^{-2}	4.5654×10^{-2}	4.4136×10^{-2}	1.5334×10^{-3}	1.5234×10^{-5}
0.9	5.0	5.3064×10^{-2}	5.6860×10^{-2}	5.3175×10^{-2}	3.7960×10^{-3}	1.1051×10^{-4}
1.0	6.0	1.6932×10^{-2}	1.9454×10^{-2}	1.9536×10^{-2}	2.5228×10^{-3}	2.6044×10^{-3}

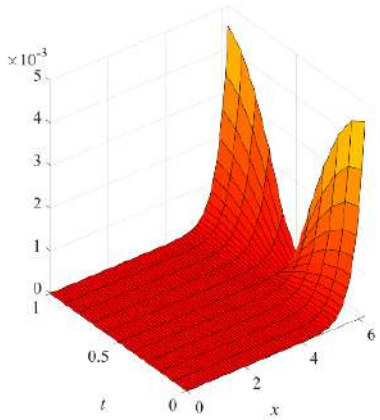
Table 1. Comparison of the real parts of exact solution ψ_e in (47), approximate solutions $\psi_{7(45)}$, $\psi_{7(49)}$ in (45), (49) for $n = 7$ and absolute errors at $y, z = 0.25$ in Example 1.



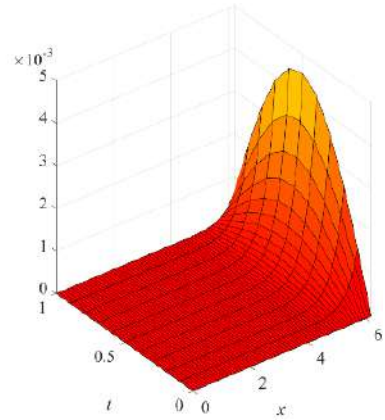
(a) $\left|\mathrm{Re}\left[\psi_e - \psi_{7(45)}\right]\right|$



(b) $\left|\mathrm{Im}\left[\psi_e - \psi_{7(45)}\right]\right|$



(c) $\left|\mathrm{Re}\left[\psi_e - \psi_{7(49)}\right]\right|$



(d) $\left|\mathrm{Im}\left[\psi_e - \psi_{7(49)}\right]\right|$

Figure 2. The graphs of the real and imaginary parts absolute errors of exact solution u_e in (47) and approximate solutions $u_{7(45)}$, $u_{7(49)}$ in (45), (49) for $n = 7$ at $y, z = 0.25$ in Example 1.

t	x	Exact solution	SVIM solution		Absolute error	
		$\mathrm{Im}[\psi_e]$	$\mathrm{Im}[\psi_{7(45)}]$	$\mathrm{Im}[\psi_{7(49)}]$	$\lvert\mathrm{Im}[\psi_e - \psi_{7(45)}]\rvert$	$\lvert\mathrm{Im}[\psi_e - \psi_{7(49)}]\rvert$
0.1	0.5	-8.6720×10^{-3}	-8.6720×10^{-3}	-8.6720×10^{-3}	0	0
0.2	1.0	-2.9082×10^{-2}	-2.9082×10^{-2}	-2.9082×10^{-2}	0	0
0.3	1.5	-4.7826×10^{-2}	-4.7826×10^{-2}	-4.7826×10^{-2}	0	0
0.4	2.0	-5.1874×10^{-2}	-5.1874×10^{-2}	-5.1874×10^{-2}	0	0
0.5	2.5	-3.6540×10^{-2}	-3.6536×10^{-2}	-3.6540×10^{-2}	3.8025×10^{-6}	0
0.6	3.0	-8.4119×10^{-3}	-8.4073×10^{-3}	-8.4119×10^{-3}	4.5854×10^{-6}	0
0.7	4.0	3.9986×10^{-2}	3.9889×10^{-2}	3.9989×10^{-2}	9.7440×10^{-5}	2.4370×10^{-6}
0.8	4.5	4.0415×10^{-2}	4.0002×10^{-2}	4.0429×10^{-2}	4.1360×10^{-4}	1.3955×10^{-5}
0.9	5.0	2.5085×10^{-2}	2.3929×10^{-2}	2.5137×10^{-2}	1.1554×10^{-3}	5.2242×10^{-5}
1.0	6.0	2.4135×10^{-3}	1.5576×10^{-3}	2.7848×10^{-3}	8.5596×10^{-4}	3.7125×10^{-4}

Table 2. Compa1rison of the imaginary parts of exact solution ψ_e in (47), approximate solutions $\psi_{7(45)}$, $\psi_{7(49)}$ in (45), (49) for $n = 7$ and absolute errors at $y, z = 0.25$ in Example 1.

Example 2. We consider the following 3D nonlinear Schrödinger equation

$$\left(i\frac{\partial}{\partial t} + \nabla^2 - V(x,y,z) + 2|\psi(x,y,z,t)|^2\right)\psi(x,y,z,t) = 0\ ,\ t > 0\ ,\ x, y, z \in [0,2\pi) \quad (50)$$

with potential

$$V(x,y,z) = -2(1 - \sin^2 x \sin^2 y \sin^2 z) \quad (51)$$

and conditions

$$\psi(x,y,z,0) = \sin x \sin y \sin z\ , \quad (52)$$

$$\psi(0,y,z,t) = 0\ ,\ \psi_x(0,y,z,t) = \sin y \sin z\, e^{-it}\ ,$$

$$\psi(x,0,z,t) = 0\ ,\ \psi_y(x,0,z,t) = \sin x \sin z\, e^{-it}\ , \quad (53)$$

$$\psi(x,y,0,t) = 0\ ,\ \psi_z(x,y,0,t) = \sin x \sin y\, e^{-it}\ .$$

Using the Shehu variational iteration method in the problem (50)-(53), we obtain the following iteration

$$\psi_{n+1}(x,y,z,t) = \sin x \sin y \sin z + S^{-1}\left[i\frac{u}{s}S[(\nabla^2 + 2(1-\sin^2 x \sin^2 y \sin^2 z) + 2|\psi_n(x,y,z,t)|^2)\psi_n(x,y,z,t)]\right]. \quad (54)$$

$\psi_0(x,y,z,t) = \sin x \sin y \sin z$.

In cases $n = 1,2,3$, we get following approximations,

$\psi_1(x,y,z,t) = \sin x \sin y \sin z\,(1+it)$

$\psi_2(x,y,z,t) = \sin x \sin y \sin z\left(1+it-\frac{t^2}{2!}\right)$

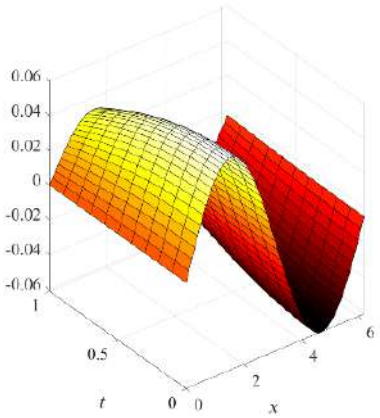
$\psi_3(x,y,z,t) = \sin x \sin y \sin z\left(1+it-\frac{t^2}{2!}-i\frac{t^3}{2!}\right)$

We continue in this manner to obtain the approximation to order n

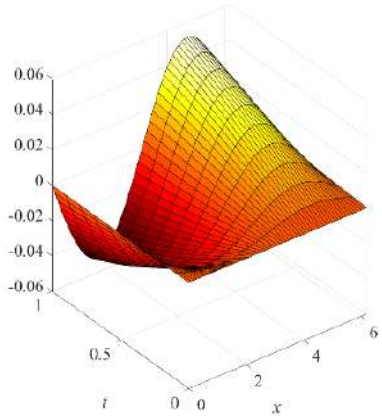
$$\psi_n(x,y,z,t) = \sin x \sin y \sin z\left(1+it-\frac{t^2}{2!}-i\frac{t^3}{2!}+\cdots+\frac{(it)^n}{n!}\right) \quad (55)$$

Exact solution is

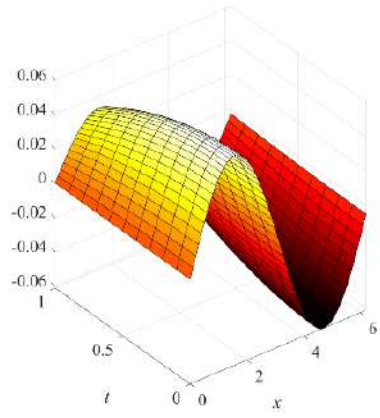
$$\psi(x,y,z,t) = \sin x \sin y \sin z\, e^{-it}. \quad (56)$$



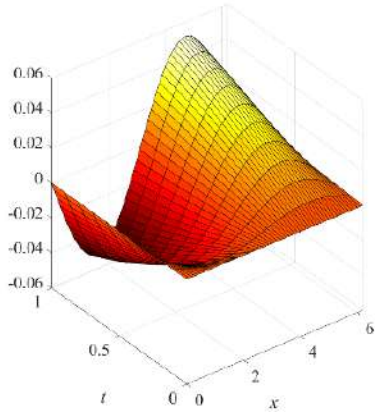
(a) Re$[\psi_e]$



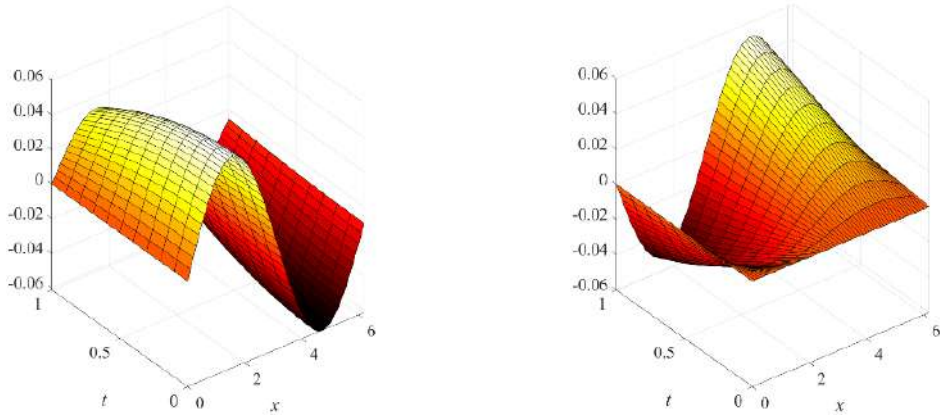
(b) Im$[\psi_e]$



(c) Re$[\psi_{7(55)}]$



(d) Im$[\psi_{7(55)}]$



(e) $\mathrm{Re}[\psi_{7(58)}]$ **(f)** $\mathrm{Im}[\psi_{7(58)}]$

Figure 3. The graphs of the real and imaginary parts of exact solution u_e in (56) and approximate solutions $u_{7(55)}$, $u_{7(58)}$ in (55), (58) for $n = 7$ at $y, z = 0.25$ in Example 2.

Using the following iteration formula,

$$\psi_{n+1}(x,y,z,t) = xe^{-it}\sin y \sin z + S^{-1}\left[-\frac{s^2}{u^2}S\left[\left(i\frac{\partial}{\partial t} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + 2(1-\sin^2 x \sin^2 y \sin^2 z) + |\psi_n(x,y,z,t)|^2)\psi_n(x,y,z,t)\right]\right],$$

$n \geq 0.$ (57)

and

$$\psi_0(x,y,z,t) = xe^{-it}\sin y \sin z \quad (58)$$

we get

$$\psi_1(x,y,z,t) = \left(x - \frac{x^3}{3!}\right)e^{-it}\sin y \sin z$$

$$\psi_2(x,y,z,t) = \left(x - \frac{x^3}{3!} + \frac{x^5}{5!}\right)e^{-it}\sin y \sin z$$

$$\psi_3(x,y,z,t) = \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}\right)e^{-it}\sin y \sin z$$

We continue in this manner to obtain the approximation to order n

$$\psi_n(x,y,z,t) = \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots + (-1)^n\frac{x^{2n+1}}{(2n+1)!}\right)e^{-it}\sin y \sin z \quad (58)$$

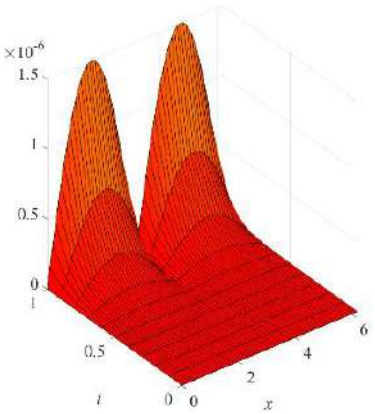
Therefore, the solution is

$$\psi(x,y,z,t) = \sin x \sin y \sin z\, e^{-it}. \quad (59)$$

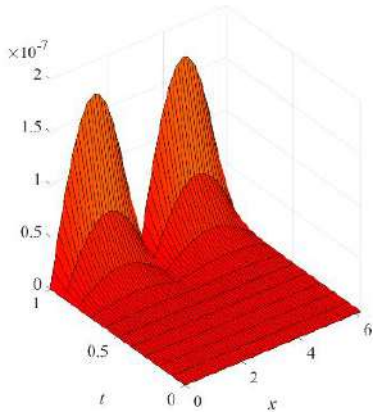
t	x	Exact solution $\mathrm{Re}[\psi_e]$	SVIM solution $\mathrm{Re}[\psi_{7(55)}]$	SVIM solution $\mathrm{Re}[\psi_{7(58)}]$	Absolute error $\lvert\mathrm{Re}[\psi_e - \psi_{7(55)}]\rvert$	Absolute error $\lvert\mathrm{Re}[\psi_e - \psi_{7(58)}]\rvert$
0.1	0.5	2.919842×10^{-2}	2.919842×10^{-2}	2.919842×10^{-2}	0	0

0.2	1.0	5.047868×10^{-2}	5.047868×10^{-2}	5.047868×10^{-2}	0	0
0.3	1.5	5.832844×10^{-2}	5.832844×10^{-2}	5.832844×10^{-2}	0	0
0.4	2.0	5.126343×10^{-2}	5.126343×10^{-2}	5.126343×10^{-2}	0	0
0.5	2.5	3.214735×10^{-2}	3.214735×10^{-2}	3.214735×10^{-2}	0	0
0.6	3.0	7.129063×10^{-3}	7.129060×10^{-3}	7.129045×10^{-3}	3.584000×10^{-9}	1.786900×10^{-8}
0.7	4.0	-3.542972×10^{-2}	-3.542965×10^{-2}	-3.543188×10^{-2}	6.587000×10^{-8}	2.159300×10^{-6}
0.8	4.5	-4.168631×10^{-2}	-4.168606×10^{-2}	-4.170070×10^{-2}	2.472099×10^{-7}	1.439345×10^{-5}
0.9	5.0	-3.648510×10^{-2}	-3.648448×10^{-2}	-3.656109×10^{-2}	6.210300×10^{-7}	7.598427×10^{-5}
1.0	6.0	-9.240609×10^{-3}	-9.240190×10^{-3}	-1.066200×10^{-3}	4.194960×10^{-7}	1.421395×10^{-3}

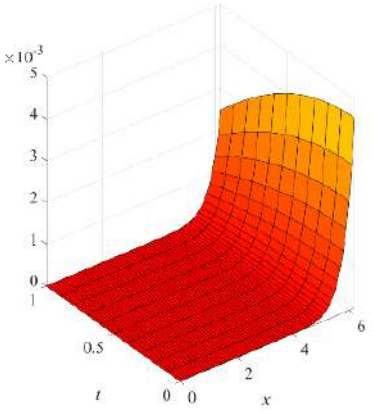
Table 3. Comparison of the real parts of exact solution ψ_e in (56), approximate solutions $\psi_{7(55)}$, $\psi_{7(58)}$ in (55), (58) for $n = 7$ and absolute errors at $y, z = 0.25$ in Example 2.



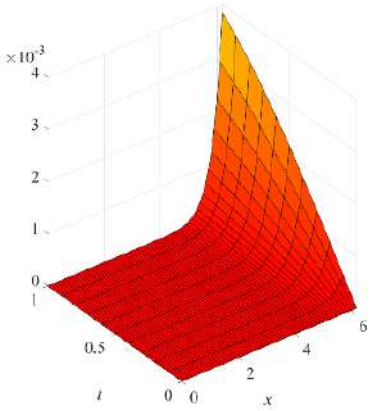
(a) $\left|\mathrm{Re}\left[\psi_e - \psi_{7(55)}\right]\right|$



(b) $\left|\mathrm{Im}\left[\psi_e - \psi_{7(55)}\right]\right|$



(c) $\left|\mathrm{Re}\left[\psi_e - \psi_{7(58)}\right]\right|$



(d) $\left|\mathrm{Im}\left[\psi_e - \psi_{7(58)}\right]\right|$

Figure 4. The graphs of the real and imaginary parts absolute errors of exact solution u_e in (56) and approximate solutions $u_{7(55)}$, $u_{7(58)}$ in (55), (58) for $n = 7$ at $y, z = 0.25$ in Example 2.

t	x	Exact solution	SVIM solution		Absolute error	
		$\mathrm{Im}[\psi_e]$	$\mathrm{Im}[\psi_{7(55)}]$	$\mathrm{Im}[\psi_{7(58)}]$	$\lvert\mathrm{Im}[\psi_e - \psi_{7(55)}]\rvert$	$\lvert\mathrm{Im}[\psi_e - \psi_{7(58)}]\rvert$
0.1	0.5	-2.929614×10^{-3}	-2.929614×10^{-3}	-2.929614×10^{-3}	0	0
0.2	1.0	-1.023254×10^{-2}	-1.023254×10^{-2}	-1.023254×10^{-2}	0	0
0.3	1.5	-1.804310×10^{-2}	-1.804310×10^{-2}	-1.804310×10^{-2}	0	0
0.4	2.0	-2.167383×10^{-2}	-2.167383×10^{-2}	-2.167383×10^{-2}	0	0
0.5	2.5	-1.756218×10^{-2}	-1.756218×10^{-2}	-1.756218×10^{-2}	0	0
0.6	3.0	-4.877255×10^{-3}	-4.877254×10^{-3}	-4.877242×10^{-3}	2.389997×10^{-10}	1.222500×10^{-8}
0.7	4.0	2.984204×10^{-2}	2.984203×10^{-2}	2.984386×10^{-2}	5.129945×10^{-9}	1.818760×10^{-6}
0.8	4.5	4.292183×10^{-2}	4.292181×10^{-2}	4.293665×10^{-2}	2.200011×10^{-8}	1.482005×10^{-5}
0.9	5.0	4.597700×10^{-2}	4.597694×10^{-2}	4.607275×10^{-2}	6.220982×10^{-8}	9.575220×10^{-5}
1.0	6.0	1.439140×10^{-2}	1.439135×10^{-2}	1.660509×10^{-2}	4.970999×10^{-8}	2.213691×10^{-3}

Table 4. Comparison of the imaginary parts of exact solution ψ_e in (56), approximate solutions $\psi_{7(55)}$, $\psi_{7(58)}$ in (55), (58) for $n = 7$ and absolute errors at $y, z = 0.25$ in Example 2.

Conclusion

In this study, approximate and exact solutions of the three-dimensional linear and non-linear Schrödinger equation is obtained using the Shehu variational iteration method. The accuracy of the approximate and exact solutions of the method is given with numerical examples. With the help of graphics, the imaginary and real parts of the exact solution and approximate solutions are compared, and absolute error graphs are included. With the compatibility of the results, it has been observed that the method provides rapid approximations to the exact solution.

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Approximate Solutions of Schrödinger Equations by Laplace Variational Iteration Method

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Abstract: Schrödinger equations have important applications in various branches of physics and engineering. In the present research, three dimensional linear and nonlinear time dependent Schrödinger equations are investigated. Laplace variational iteration method, based on the Laplace transform and variational iteration method, is presented for approximate solutions of three dimensional linear and nonlinear time dependent Schrödinger equations. Different correctional functionals are constructed and their Lagrange multipliers are determined. With these Lagrange multipliers, iteration formulas are formed. To illustrate the efficiency of the Laplace variational iteration method, example problems for three dimensional linear and nonlinear time dependent Schrödinger equations are considered. Two different iteration formulas are used for every example problem. Approximate solutions are compared with exact solutions in graphical form. From the numerical results, it is seen that the Laplace variational iteration method is efficient in searching for approximate solutions of the three dimensional linear and nonlinear time dependent Schrödinger equations. This method can be successfully applied to other linear and nonlinear equations.

Keywords: Laplace transform, variational iteration method, Schrödinger equation.

1 Introduction

The time dependent Schrödinger equation is

$$\left(i\hbar\frac{\partial}{\partial t}+\frac{\hbar^2}{2m}\nabla^2-V(x,y,z)-\kappa|\psi(x,y,z,t)|^2\right)\psi(x,y,z,t)=0\ ,\ t>0\ ,\ x,y,z\in\mathbb{R} \qquad (1)$$

Here, $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$ is the Laplace operator, κ is the real parameter, m is the mass of the particle and $\hbar=\frac{h}{2\pi}$ where $h=6.62606896(33)\times10^{-34}$ Js is the Planck constant. $\psi\in L^2$ is the solution function or wave function, V is the arbitrary potential function [1, 2]. In this study, three dimensional linear and nonlinear time dependent Schrödinger equations are investigated by Laplace variational iteration method (LVIM).

In 1997, Chinese mathematician J.H. He developed the variational iteration method(VIM) to solve nonlinear differential equations [3, 4]. VIM requires the identification of a general

Lagrange multiplier. To identify easily the Lagrange multiplier, Laplace transform was adopted in VIM [5-7]. LVIM has been used to find the solutions of various problems. For example, ordinary differential equations [8], Smoluchowski coagulation equation [9], fractional heat equations [10], Kuramoto–Sivashinsky equations [11], two-dimensional linear Schrödinger equations [12].

In this work, we apply LVIM to find the solutions of three dimensional linear and nonlinear time dependent Schrödinger equations. Different iteration formulas are formed for obtaing approximate solutions. Example problems are considered to illustrate the efficiency of LVIM.

2 Preliminaries

Definition 2.1 [13] Let u be a function of t defined for all $t \geq 0$. Then the Laplace transform of u, is defined as

$$F(s) = \mathcal{L}[u(t)] = \int_0^\infty e^{-st} u(t) dt \tag{2}$$

where s is the parameter.

Theorem 2.1 (Linearity property). [13] Let u, w be two functions of t defined for all $t \geq 0$. Then

$$\mathcal{L}[au(t) + bw(t)] = a\mathcal{L}[u(t)] + b\mathcal{L}[w(t)] \tag{3}$$

where a and b are constants.

Theorem 2.2 (Laplace transform of derivatives). [13] If $F(s) = \mathcal{L}[u(t)]$, then

$$\mathcal{L}[u'(t)] = sF(s) - u(0) \tag{4}$$

$$\mathcal{L}[u''(t)] = s^2F(s) - su(0) - u'(0) \tag{5}$$

$$\vdots$$

$$\mathcal{L}\left[\frac{d^n u(t)}{dt^n}\right] = s^nF(s) - s^{n-1}u(0) - s^{n-2}u'(0) - s^{n-3}u''(0) - \cdots - su^{(n-2)}(0) - u^{(n-1)}(0). \tag{6}$$

Definition 2.2 [13] If $\mathcal{L}[u(t)] = F(s)$, then the inverse Laplace transform is defined as

$$\mathcal{L}^{-1}[F(s)] = u(t)\ ,\ t \geq 0. \tag{7}$$

Theorem 2.3 (Linearity property). [13] Let u, w be two functions of t defined for all $t \geq 0$. Let F, G be the functions of s such that $F(s) = \mathcal{L}[u(t)]$, $G(s) = \mathcal{L}[w(t)]$. Then

$$\mathcal{L}^{-1}[cF(s) + dG(s)] = c\mathcal{L}^{-1}[F(s)] + d\mathcal{L}^{-1}[G(s)] \tag{8}$$

where c and d are constants.

Definition 2.3 [13] Let u, w be integrable functions, then the convolution of u, w is defined as

$$u(t) * w(t) = \int_0^t u(t-\varepsilon)w(\varepsilon)d\varepsilon. \quad (9)$$

Theorem 2.4 (Convolution Theorem). [13] Let u, w be integrable functions, then

$$\mathcal{L}[u(t) * w(t)] = \mathcal{L}[u(t)]\mathcal{L}[w(t)]. \quad (10)$$

3 Laplace Variational Iteration Method

LVIM is a combination of the Laplace transform and VIM [3-7]. Consider the differential equation

$$(L+N)u(t) = g(t) \quad (11)$$

where L is a linear operator, N is a nonlinear operator and g is a known analytical function. The correction functional for (11) given by

$$u_{n+1}(t) = u_n(t) + \int_0^t \lambda(\varepsilon)\big(Lu_n(\varepsilon) + N\widetilde{u_n}(\varepsilon) - g(\varepsilon)\big)d\varepsilon\,, n \geq 0 \quad (12)$$

where λ is a general Lagrange multiplier, which can be identified optimally via the variational theory. $n \geq 0$, u_n is the nth approximation. $\widetilde{u_n}$ is a restricted variation, that is, $\delta\widetilde{u_n} = 0$. The general form of Lagrange multiplier is found to be of the form

$$\lambda = \lambda(t-\varepsilon). \quad (13)$$

Applying the Laplace transform on both sides of (12), the correction functional will be transformed as follows

$$\mathcal{L}[u_{n+1}(t)] = \mathcal{L}[u_n(t)] + \mathcal{L}\left[\int_0^t \lambda(t-\varepsilon)\big(Lu_n(\varepsilon) + N\widetilde{u_n}(\varepsilon) - g(\varepsilon)\big)d\varepsilon\right] \quad (14)$$

$n \geq 0$. Therefore

$$\begin{aligned}\mathcal{L}[u_{n+1}(t)] &= \mathcal{L}[u_n(t)] + \mathcal{L}\big[\lambda(t) * \big(Lu_n(t) + N\widetilde{u_n}(t) - g(t)\big)\big] \\ &= \mathcal{L}[u_n(t)] + \mathcal{L}[\lambda(t)]\mathcal{L}[Lu_n(t) + N\widetilde{u_n}(t) - g(t)]. \quad (15)\end{aligned}$$

Taking the variation with respect to u_n of both sides of (15), leads to

$$\mathcal{L}[\delta u_{n+1}(t)] = \mathcal{L}[\delta u_n(t)] + \delta\mathcal{L}[\lambda(t)]\mathcal{L}[Lu_n(t)]. \quad (16)$$

The stationary condition is constructed from (16). From stationary condition, λ is determined. Substituting λ into (14) and applying inverse Laplace transform on both sides of (14) gives the iteration formula. Approximate solutions u_n are obtained using any selective function u_0 and iteration formula. The solution is

$$u(t) = \lim_{n\to\infty} u_n(t). \quad (17)$$

4 Application of LVIM to the linear Schrödinger equation

For $\kappa = 0$, $\hbar = 2m = 1$, (1) becomes

$$\left(i\frac{\partial}{\partial t}+\nabla^2-V(x,y,z)\right)\psi(x,y,z,t)=0\ ,\ t>0\ ,\ x,\ y,\ z\in\mathbb{R} \tag{18}$$

The correction functionals for (18) are constructed in the following

$$\psi_{n+1}(x,y,z,t)=\psi_n(x,y,z,t)+\int_0^t\lambda_1(t-\varepsilon)\left(i\frac{\partial\psi_n(x,y,z,\varepsilon)}{\partial\varepsilon}+\left(\nabla^2-V(x,y,z)\right)\widetilde{\psi_n}(x,y,z,\varepsilon)\right)d\varepsilon, \tag{19}$$

$$\psi_{n+1}(x,y,z,t)=\psi_n(x,y,z,t)+\int_0^x\lambda_2(x-\zeta)\left(\frac{\partial^2\psi_n(\zeta,y,z,t)}{\partial\zeta^2}+\left(i\frac{\partial}{\partial t}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-V(\zeta,y,z)\right)\widetilde{\psi_n}(\zeta,y,z,t)\right)d\zeta \tag{20}$$

$n\geq 0$. Here, λ_1 and λ_2 are Lagrange multipliers. Applying the Laplace transform on both sides of (19)-(20), we transform the correction functionals as follows

$$\mathcal{L}[\psi_{n+1}(x,y,z,t)]=\mathcal{L}[\psi_n(x,y,z,t)]+\mathcal{L}\left[\int_0^t\lambda_1(t-\varepsilon)\left(i\frac{\partial\psi_n(x,y,z,\varepsilon)}{\partial\varepsilon}+\left(\nabla^2-V(x,y,z)\right)\widetilde{\psi_n}(x,y,z,\varepsilon)\right)d\varepsilon\right], \tag{21}$$

$$\mathcal{L}[\psi_{n+1}(x,y,z,t)]=\mathcal{L}[\psi_n(x,y,z,t)]+\mathcal{L}\left[\int_0^x\lambda_2(x-\zeta)\left(\frac{\partial^2\psi_n(\zeta,y,z,t)}{\partial\zeta^2}+\left(i\frac{\partial}{\partial t}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-V(\zeta,y,z)\right)\widetilde{\psi_n}(\zeta,y,z,t)\right)d\zeta\right] \tag{22}$$

$n\geq 0$. When we apply the properties of the Laplace transform, we get

$$\begin{aligned}\mathcal{L}[\psi_{n+1}(x,y,z,t)]&=\mathcal{L}[\psi_n(x,y,z,t)]+\mathcal{L}\left[\lambda_1(t)*\left(i\frac{\partial\psi_n(x,y,z,t)}{\partial t}+\left(\nabla^2-V(x,y,z)\right)\widetilde{\psi_n}(x,y,z,t)\right)\right]\\&=\mathcal{L}[\psi_n(x,y,z,t)]+\mathcal{L}[\lambda_1(t)]\mathcal{L}\left[i\frac{\partial\psi_n(x,y,z,t)}{\partial t}+\left(\nabla^2-V(x,y,z)\right)\widetilde{\psi_n}(x,y,z,t)\right]\\&=\mathcal{L}[\psi_n(x,y,z,t)]+\mathcal{L}[\lambda_1(t)]\Big(i\big(s\mathcal{L}[\psi_n(x,y,z,t)]-\psi_n(x,y,z,0)\big)\\&\quad+\mathcal{L}\big[\left(\nabla^2-V(x,y,z)\right)\widetilde{\psi_n}(x,y,z,t)\big]\Big),\end{aligned} \tag{23}$$

$$\begin{aligned}\mathcal{L}[\psi_{n+1}(x,y,z,t)]&=\mathcal{L}[\psi_n(x,y,z,t)]+\mathcal{L}\left[\lambda_2(x)*\left(\frac{\partial^2\psi_n(x,y,z,t)}{\partial x^2}+\left(i\frac{\partial}{\partial t}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-V(x,y,z)\right)\widetilde{\psi_n}(x,y,z,t)\right)\right]\\&=\mathcal{L}[\psi_n(x,y,z,t)]+\mathcal{L}[\lambda_2(x)]\mathcal{L}\left[\frac{\partial^2\psi_n(x,y,z,t)}{\partial x^2}\right.\end{aligned}$$

$$+\left(i\frac{\partial}{\partial t}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-V(x,y,z)\right)\widetilde{\psi_n}(x,y,z,t)\Bigg)\Bigg]$$

$$=\mathcal{L}[\psi_n(x,y,z,t)]+\mathcal{L}[\lambda_2(x)](s^2\mathcal{L}[\psi_n(x,y,z,t)]-s\psi_n(0,y,z,t)$$

$$-\frac{\partial\psi_n(x,y,z,t)}{\partial x}\Big|_{x=0}+\mathcal{L}\left[\left(i\frac{\partial}{\partial t}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-V(x,y,z)\right)\widetilde{\psi_n}(x,y,z,t)\right]\Bigg) \quad (24)$$

$n\geq 0$. Taking the variation with respect to ψ_n of both sides of (23)-(24) with known $\delta\psi_n=0$, leads to

$$\mathcal{L}[\delta\psi_{n+1}(x,y,z,t)]=(1+is\mathcal{L}[\lambda_1(t)])\mathcal{L}[\delta\psi_n(x,y,z,t)], \quad (25)$$

$$\mathcal{L}[\delta\psi_{n+1}(x,y,z,t)]=(1+s^2\mathcal{L}[\lambda_2(x)])\mathcal{L}[\delta\psi_n(x,y,z,t)] \quad (26)$$

$n\geq 0$. Then, we have the following stationary conditions

$$\mathcal{L}[\lambda_1(t)]=\frac{i}{s}\ ,\ \mathcal{L}[\lambda_2(x)]=-\frac{1}{s^2}. \quad (27)$$

Therefore

$$\lambda_1(t)=i\ ,\ \lambda_2(x)=-x. \quad (28)$$

Substituting (28) into (21)-(22) results in the following iterative schemes

$$\mathcal{L}[\psi_{n+1}(x,y,z,t)]=\mathcal{L}[\psi_n(x,y,z,t)]+\mathcal{L}[i]\mathcal{L}\left[i\frac{\partial\psi_n(x,y,z,t)}{\partial t}+\big(\nabla^2-\right.$$

$$\left.V(x,y,z)\big)\psi_n(x,y,z,t)\right], \quad (29)$$

$$\mathcal{L}[\psi_{n+1}(x,y,z,t)]=\mathcal{L}[\psi_n(x,y,z,t)]+\mathcal{L}[-x]\mathcal{L}\left[\frac{\partial^2\psi_n(x,y,z,t)}{\partial x^2}\right.$$

$$\left.+\left(i\frac{\partial}{\partial t}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-V(x,y,z)\right)\psi_n(x,y,z,t)\right] \quad (30)$$

$n\geq 0$. By applying inverse Laplace transform on both sides of (29)-(30), we get the following iteration formulas:

$$\psi_{n+1}(x,y,z,t)=\psi_n(x,y,z,0)+\mathcal{L}^{-1}\left[\frac{i}{s}\mathcal{L}\left[\big(\nabla^2-V(x,y,z)\big)\psi_n(x,y,z,t)\right]\right], \quad (31)$$

$$\psi_{n+1}(x,y,z,t)=\psi_n(0,y,z,t)+x\frac{\partial\psi_n(x,y,z,t)}{\partial x}\Big|_{x=0}$$

$$+\mathcal{L}^{-1}\left[-\frac{1}{s^2}\mathcal{L}\left[\left(i\frac{\partial}{\partial t}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-V(x,y,z)\right)\psi_n(x,y,z,t)\right]\right] \quad (32)$$

$n\geq 0$. Similarly,

$$\psi_{n+1}(x,y,z,t)=\psi_n(x,0,z,t)+y\frac{\partial\psi_n(x,y,z,t)}{\partial y}\Big|_{y=0}$$

$$+\mathcal{L}^{-1}\left[-\frac{1}{s^2}\mathcal{L}\left[\left(i\frac{\partial}{\partial t}+\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}-V(x,y,z)\right)\psi_n(x,y,z,t)\right]\right], \quad (33)$$

$$\psi_{n+1}(x,y,z,t)=\psi_n(x,y,0,t)+z\frac{\partial\psi_n(x,y,z,t)}{\partial z}\Big|_{z=0}$$

$$+\mathcal{L}^{-1}\left[-\frac{1}{s^2}\mathcal{L}\left[\left(i\frac{\partial}{\partial t}+\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}-V(x,y,z)\right)\psi_n(x,y,z,t)\right]\right] \quad (34)$$

$n \geq 0$. The solution is

$$\psi(x,y,z,t)=\lim_{n\to\infty}\psi_n(x,y,z,t). \quad (35)$$

5 Application of LVIM to the nonlinear Schrödinger equation

For $\hbar = 2m = 1$, (1) becomes

$$\left(i\frac{\partial}{\partial t}+\nabla^2-V(x,y,z)-\kappa|\psi(x,y,z,t)|^2\right)\psi(x,y,z,t)=0\ ,\ t>0\ ,\ x,\ y,\ z\in\mathbb{R} \quad (36)$$

The correction functionals for (36) are constructed in the following

$$\psi_{n+1}(x,y,z,t)=\psi_n(x,y,z,t)+\int_0^t\lambda_1(t-\varepsilon)\left(i\frac{\partial\psi_n(x,y,z,\varepsilon)}{\partial\varepsilon}+(\nabla^2-V(x,y,z)\right.$$
$$\left.-\kappa\left|\widetilde{\psi_n}(x,y,z,t)\right|^2\right)\widetilde{\psi_n}(x,y,z,\varepsilon)\Big)\,d\varepsilon, \quad (37)$$

$$\psi_{n+1}(x,y,z,t)=\psi_n(x,y,z,t)+\int_0^x\lambda_2(x-\zeta)\left(\frac{\partial^2\psi_n(\zeta,y,z,t)}{\partial\zeta^2}\right.$$
$$\left.+\left(i\frac{\partial}{\partial t}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-V(\zeta,y,z)-\kappa\left|\widetilde{\psi_n}(x,y,z,t)\right|^2\right)\widetilde{\psi_n}(\zeta,y,z,t)\right)d\zeta \quad (38)$$

$n \geq 0$. Here, λ_1 and λ_2 are Lagrange multipliers. Applying the Laplace transform on both sides of (37)-(38) and using the properties of the Laplace transform, we get

$$\mathcal{L}[\psi_{n+1}(x,y,z,t)]=\mathcal{L}[\psi_n(x,y,z,t)]+\mathcal{L}[\lambda_1(t)]\Big(i\big(s\mathcal{L}[\psi_n(x,y,z,t)]-\psi_n(x,y,z,0)\big)$$
$$+\mathcal{L}\left[\left(\nabla^2-V(x,y,z)-\kappa\left|\widetilde{\psi_n}(x,y,z,t)\right|^2\right)\widetilde{\psi_n}(x,y,z,t)\right]\Big), \quad (39)$$

$$\mathcal{L}[\psi_{n+1}(x,y,z,t)]=\mathcal{L}[\psi_n(x,y,z,t)]+\mathcal{L}[\lambda_2(x)]\Big(i\big(s^2\mathcal{L}[\psi_n(x,y,z,t)]-s\psi_n(0,y,z,t)\big)$$
$$-\frac{\partial\psi_n(x,y,z,t)}{\partial x}\Big|_{x=0}\Big)+\mathcal{L}\left[\left(i\frac{\partial}{\partial t}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-V(x,y,z)-\right.\right.$$
$$\left.\left.\kappa\left|\widetilde{\psi_n}(x,y,z,t)\right|^2\right)\widetilde{\psi_n}(x,y,z,t)\right]\Big) \quad (40)$$

$n \geq 0$. We take the variation with respect to ψ_n of both sides of (39)-(40). Then, the Lagrange multipliers are obtained as

$$\lambda_1(t)=i\ ,\ \lambda_2(x)=-x. \quad (41)$$

Substituting (41) into (37)-(38) results in the following iterative schemes

$$\mathcal{L}[\psi_{n+1}(x,y,z,t)]=\mathcal{L}[\psi_n(x,y,z,t)]+\mathcal{L}[i]\mathcal{L}\left[i\frac{\partial\psi_n(x,y,z,t)}{\partial t}\right.$$
$$\left.+(\nabla^2-V(x,y,z)-\kappa|\psi_n(x,y,z,t)|^2)\psi_n(x,y,z,t)\big)\right], \quad (42)$$

$$\mathcal{L}[\psi_{n+1}(x,y,z,t)]=\mathcal{L}[\psi_n(x,y,z,t)]+\mathcal{L}[-x]\mathcal{L}\left[\frac{\partial^2\psi_n(x,y,z,t)}{\partial x^2}\right.$$

$$+\left(i\frac{\partial}{\partial t}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-V(x,y,z)-\right.$$

$$\left.\kappa|\psi_n(x,y,z,t)|^2\right)\psi_n(x,y,z,t)\Big)\Big] \tag{43}$$

$n \geq 0$. By applying inverse Laplace transform on both sides of (42)-(43), we get the following iteration formulas:

$$\psi_{n+1}(x,y,z,t)=\psi_n(x,y,z,0)+\mathcal{L}^{-1}\left[\frac{i}{s}\mathcal{L}[(\nabla^2-V(x,y,z)-\right.$$

$$\left.\kappa|\psi_n(x,y,z,t)|^2)\psi_n(x,y,z,t)]\right], \tag{44}$$

$$\psi_{n+1}(x,y,z,t)=\psi_n(0,y,z,t)+x\frac{\partial\psi_n(x,y,z,t)}{\partial x}\Big|_{x=0}$$

$$+\mathcal{L}^{-1}\left[-\frac{1}{s^2}\mathcal{L}\left[\left(i\frac{\partial}{\partial t}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-V(x,y,z)-\right.\right.\right.$$

$$\left.\left.\left.\kappa|\psi_n(x,y,z,t)|^2\right)\psi_n(x,y,z,t)\right]\right] \tag{45}$$

$n \geq 0$. Similarly,

$$\psi_{n+1}(x,y,z,t)=\psi_n(x,0,z,t)+y\frac{\partial\psi_n(x,y,z,t)}{\partial y}\Big|_{y=0}$$

$$+\mathcal{L}^{-1}\left[-\frac{1}{s^2}\mathcal{L}\left[\left(i\frac{\partial}{\partial t}+\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}-V(x,y,z)-\right.\right.\right.$$

$$\left.\left.\left.\kappa|\psi_n(x,y,z,t)|^2\right)\psi_n(x,y,z,t)\right]\right], \tag{46}$$

$$\psi_{n+1}(x,y,z,t)=\psi_n(x,y,0,t)+z\frac{\partial\psi_n(x,y,z,t)}{\partial z}\Big|_{z=0}$$

$$+\mathcal{L}^{-1}\left[-\frac{1}{s^2}\mathcal{L}\left[\left(i\frac{\partial}{\partial t}+\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}-V(x,y,z)-\right.\right.\right.$$

$$\left.\left.\left.\kappa|\psi_n(x,y,z,t)|^2\right)\psi_n(x,y,z,t)\right]\right] \tag{47}$$

$n \geq 0$. The solution is

$$\psi(x,y,z,t)=\lim_{n\to\infty}\psi_n(x,y,z,t). \tag{48}$$

Example 1. Consider the following problem for the linear Schrödinger equation

$$\left(i\frac{\partial}{\partial t}+\nabla^2\right)\psi(x,y,z,t)=0\ ,\ t>0\ ,\ x,\ y,\ z\in[0,2\pi) \tag{49}$$

$$\psi(x,y,z,0)=\sin x+\sin y+\sin z\ , \tag{50}$$

$$\begin{aligned}&\psi(0,y,z,t)=(\sin y+\sin z)e^{-it}\quad,\quad \psi_x(0,y,z,t)=e^{-it}\\&\psi(x,0,z,t)=(\sin x+\sin z)e^{-it}\quad,\quad \psi_y(x,0,z,t)=e^{-it}\\&\psi(x,y,0,t)=(\sin x+\sin y)e^{-it}\quad,\quad \psi_z(x,y,0,t)=e^{-it}.\end{aligned} \tag{51}$$

Case 1. In order to obtain approximate solutions, we use iteration formula (31). The formula (31) becomes

$$\psi_{n+1}(x,y,z,t) = \sin x + \sin y + \sin z + \mathcal{L}^{-1}\left[\frac{i}{s}\mathcal{L}[\nabla^2\psi_n(x,y,z,t)]\right] \tag{52}$$

$n \geq 0$. Starting with

$$\psi_0(x,y,z,t) = \psi(x,y,z,0) \tag{53}$$

and using (52), we get

$$\psi_1(x,y,z,t) = (1-it)\psi_0(x,y,z,t)$$

$$\psi_2(x,y,z,t) = \left(1 - it - \frac{t^2}{2!}\right)\psi_0(x,y,z,t)$$

$$\vdots$$

$$\psi_n(x,y,z,t) = \left(1 - it - \frac{t^2}{2!} + \cdots + \frac{(-it)^n}{n!}\right)\psi_0(x,y,z,t). \tag{54}$$

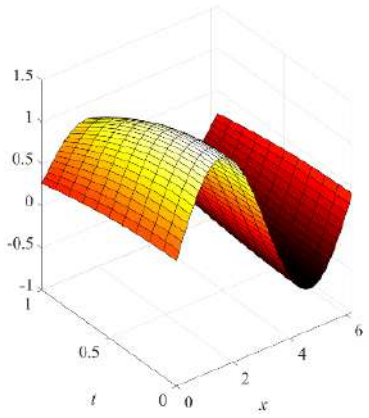
The solution is

$$\psi(x,y,z,t) = \lim_{n\to\infty}\psi_n(x,y,z,t) = \psi_e(x,y,z,t). \tag{55}$$

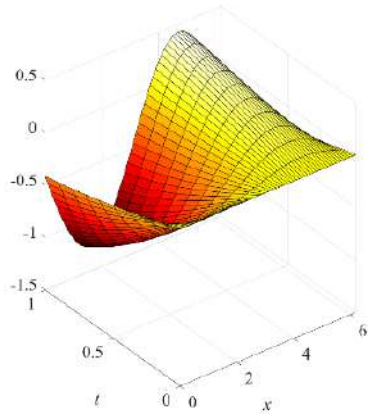
Here

$$\psi_e(x,y,z,t) = (\sin x + \sin y + \sin z)e^{-it} \tag{56}$$

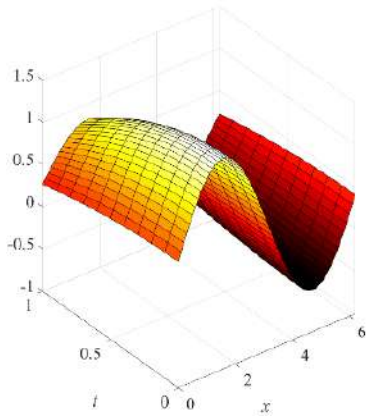
is the exact solution.

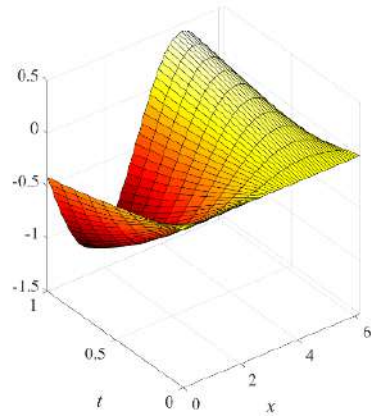


(a) $\mathrm{Re}[\psi_e]$



(b) $\mathrm{Im}[\psi_e]$





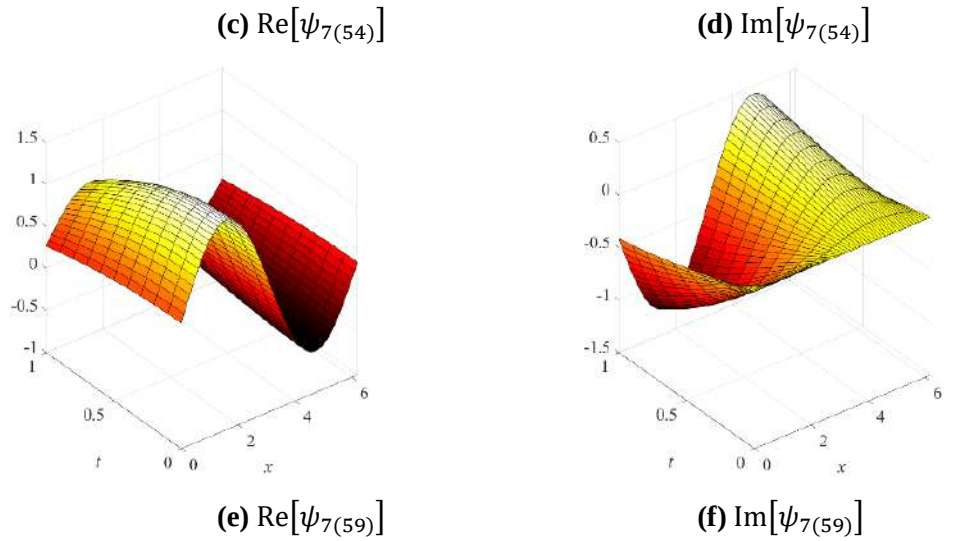


Figure 1. The graphs of the real and imaginary parts of exact solution ψ_e in (56) and approximate solutions $\psi_{7(54)}$, $\psi_{7(59)}$ in (54), (59) for $n = 7$ at $y, z = 0.25$ in Example 1.

Case 2. We use iteration formula (32). The formula (32) becomes

$$\psi_{n+1}(x,y,z,t) = (x + \sin y + \sin z)e^{-2t} + \mathcal{L}^{-1}\left[-\frac{1}{s^2}\mathcal{L}\left[\left(i\frac{\partial}{\partial t} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\psi_n(x,y,z,t)\right]\right] \quad (57)$$

$n \geq 0$. We begin with the zeroth approximation

$$\psi_0(x,y,z,t) = (x + \sin y + \sin z)e^{-it}. \quad (58)$$

By using iteration formula (57), the following approximations are obtained:

$$\psi_1(x,y,z,t) = \left(1 - \frac{x^3}{3!} + \sin y + \sin z\right)e^{-it}$$

$$\psi_2(x,y,z,t) = \left(1 - \frac{x^3}{3!} + \frac{x^5}{5!} + \sin y + \sin z\right)e^{-it}$$

$$\vdots$$

$$\psi_n(x,y,z,t) = \left(1 - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + (-1)^n\frac{x^{2n+1}}{(2n+1)!} + \sin y + \sin z\right)e^{-it}. \quad (59)$$

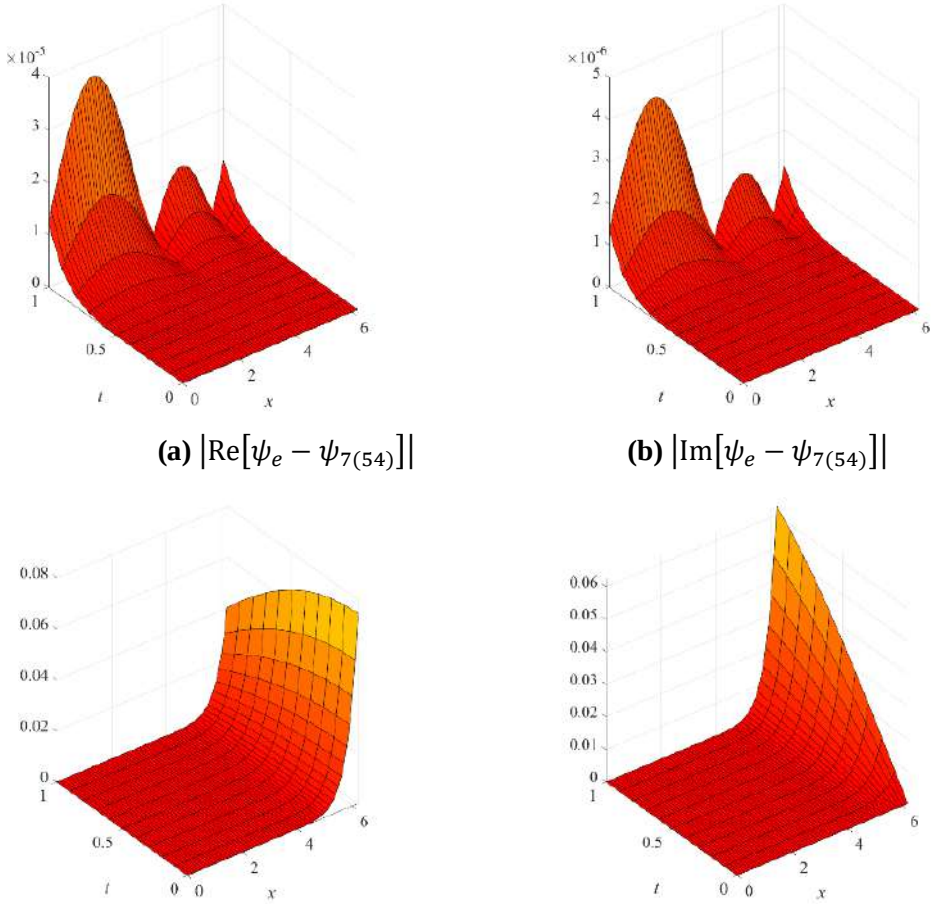
We get the following solution

$$\psi(x,y,z,t) = (\sin x + \sin y + \sin z)e^{-it}. \quad (60)$$

t	x	Exact solution	LVIM solution		Absolute error	
		$\mathrm{Re}[\psi_e]$	$\mathrm{Re}[\psi_{7(54)}]$	$\mathrm{Re}[\psi_{7(59)}]$	$\lvert\mathrm{Re}[\psi_e - \psi_{7(54)}]\rvert$	$\lvert\mathrm{Re}[\psi_e - \psi_{7(59)}]\rvert$
0.1	0.5	9.6937×10^{-1}	9.6937×10^{-1}	9.6937×10^{-1}	0	0
0.2	1.0	1.3096	1.3096	1.3096	0	0
0.3	1.5	1.4257	1.4257	1.4257	0	0
0.4	2.0	1.2933	1.2933	1.2933	0	0
0.5	2.5	9.5944×10^{-1}	9.5944×10^{-1}	9.5944×10^{-1}	0	0
0.6	3.0	5.2485×10^{-1}	5.2485×10^{-1}	5.2485×10^{-1}	0	0
0.7	4.0	-2.0038×10^{-1}	-2.0038×10^{-1}	-2.0042×10^{-1}	0	3.5278×10^{-5}
0.8	4.5	-3.3632×10^{-1}	-3.3631×10^{-1}	-3.3655×10^{-1}	1.9944×10^{-6}	2.3515×10^{-4}

0.9	5.0	-2.8850×10^{-1}	-2.8849×10^{-1}	-2.8974×10^{-1}	4.9107×10^{-6}	1.2414×10^{-3}
1.0	6.0	1.1638×10^{-1}	1.1637×10^{-1}	9.3155×10^{-2}	5.2832×10^{-6}	2.3222×10^{-2}

Table 1. Comparison of the real parts of exact solution ψ_e in (56), approximate solutions $\psi_{7(54)}$, $\psi_{7(59)}$ in (54), (59) for $n = 7$ and absolute errors at $y, z = 0.25$ in Example 1.



(a) $\left|\mathrm{Re}\left[\psi_e - \psi_{7(54)}\right]\right|$ **(b)** $\left|\mathrm{Im}\left[\psi_e - \psi_{7(54)}\right]\right|$

(c) $\left|\mathrm{Re}\left[\psi_e - \psi_{7(59)}\right]\right|$ **(d)** $\left|\mathrm{Im}\left[\psi_e - \psi_{7(59)}\right]\right|$

Figure 2. The graphs of the real and imaginary parts absolute errors of exact solution ψ_e in (56) and approximate solutions $\psi_{7(54)}$, $\psi_{7(59)}$ in (54), (59) for $n = 7$ at $y, z = 0.25$ in Example 1.

t	x	Exact solution	LVIM solution		Absolute error	
		$\mathrm{Im}[\psi_e]$	$\mathrm{Im}[\psi_{7(54)}]$	$\mathrm{Im}[\psi_{7(59)}]$	$\left\|\mathrm{Im}[\psi_e - \psi_{7(54)}]\right\|$	$\left\|\mathrm{Im}[\psi_e - \psi_{7(59)}]\right\|$
0.1	0.5	-9.726105×10^{-2}	-9.726105×10^{-2}	-9.726105×10^{-2}	0	0
0.2	1.0	-2.654776×10^{-1}	-2.654776×10^{-1}	-2.654776×10^{-1}	0	0
0.3	1.5	-4.410057×10^{-1}	-4.410057×10^{-1}	-4.410057×10^{-1}	0	0
0.4	2.0	-5.467844×10^{-1}	-5.467844×10^{-1}	-5.467844×10^{-1}	0	0
0.5	2.5	-5.241464×10^{-1}	-5.241464×10^{-1}	-5.241464×10^{-1}	0	0
0.6	3.0	-3.590719×10^{-1}	-3.590719×10^{-1}	-3.590717×10^{-1}	0	1.996998×10^{-7}
0.7	4.0	1.687815×10^{-1}	1.687815×10^{-1}	1.688113×10^{-1}	0	2.971400×10^{-5}
0.8	4.5	3.462837×10^{-1}	3.462835×10^{-1}	3.465258×10^{-1}	1.775000×10^{-7}	2.421233×10^{-4}
0.9	5.0	3.635548×10^{-1}	3.635543×10^{-1}	3.651192×10^{-1}	4.918000×10^{-7}	1.564356×10^{-3}
1.0	6.0	-1.812465×10^{-1}	-1.812459×10^{-1}	-1.450802×10^{-1}	5.882003×10^{-7}	3.616627×10^{-2}

Table 2. Comparison of the imaginary parts of exact solution ψ_e in (56), approximate solutions $\psi_{7(54)}$, $\psi_{7(59)}$ in (54), (59) for $n = 7$ and absolute errors at $y, z = 0.25$ in Example 1.

Example 2. Consider the following problem for the nonlinear Schrödinger equation

$$\left(i\frac{\partial}{\partial t}+\nabla^2+2(1-\cos^2 x\cos^2 y\cos^2 z)\right.$$
$$\left.+2|\psi(x,y,z,t)|^2\right)\psi(x,y,z,t)=0\ ,\ t>0\ ,\ x,y,z\in[0,2\pi) \tag{61}$$
$$\psi(x,y,z,0)=\cos x\cos y\cos z\ , \tag{62}$$
$$\begin{aligned}&\psi(0,y,z,t)=e^{-it}\cos y\cos z\quad ,\quad \psi_x(0,y,z,t)=0\\&\psi(x,0,z,t)=e^{-it}\cos x\cos z\quad ,\quad \psi_y(x,0,z,t)=0\\&\psi(x,y,0,t)=e^{-it}\cos x\cos y\quad ,\quad \psi_z(x,y,0,t)=0.\end{aligned} \tag{63}$$

Case 1. We use iteration formula (44). The formula (44) becomes

$$\psi_{n+1}(x,y,z,t)=\cos x\cos y\cos z+\mathcal{L}^{-1}\left[\frac{i}{s}\mathcal{L}[(\nabla^2+2(1-\cos^2 x\cos^2 y\cos^2 z)\right.$$
$$\left.+2|\psi_n(x,y,z,t)|^2)\psi_n(x,y,z,t)]\right] \tag{64}$$

$n\geq 0$. Starting with

$$\psi_0(x,y,z,t)=\psi(x,y,z,0) \tag{65}$$

and using (64), we get the following approximations

$$\psi_1(x,y,z,t)=(1-it)\psi_0(x,y,z,t)$$
$$\psi_2(x,y,z,t)=\left(1-it-\frac{t^2}{2!}\right)\psi_0(x,y,z,t)$$
$$\vdots$$
$$\psi_n(x,y,z,t)=\left(1-it-\frac{t^2}{2!}+\cdots+\frac{(-it)^n}{n!}\right)\psi_0(x,y,z,t). \tag{66}$$

The solution is

$$\psi(x,y,z,t)=\lim_{n\to\infty}\psi_n(x,y,z,t)=\psi_e(x,y,z,t) \tag{67}$$

where the exact solution is

$$\psi_e(x,y,z,t)=e^{-it}\cos x\cos y\cos z\ . \tag{68}$$

Case 2. We use iteration formula (45). The formula (45) becomes

$$\psi_{n+1}(x,y,z,t)=\cos y\cos z+\mathcal{L}^{-1}\left[-\frac{1}{s^2}\mathcal{L}\left[\left(i\frac{\partial}{\partial t}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}+2(1-\cos^2 x\cos^2 y\cos^2 z)\right.\right.\right.$$
$$\left.\left.\left.+2|\psi_n(x,y,z,t)|^2\right)\psi_n(x,y,z,t)\right]\right] \tag{69}$$

$n\geq 0$. Starting with

$$\psi_0(x,y,z,t)=\psi(x,y,z,0) \tag{70}$$

and using (69), we obtain

$$\psi_1(x,y,z,t)=\left(1-\frac{x^2}{2!}\right)\psi_0(x,y,z,t)$$
$$\psi_2(x,y,z,t)=\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}\right)\psi_0(x,y,z,t)$$
$$\vdots$$

$$\psi_n(x,y,z,t) = \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + (-1)^n \frac{x^{2n}}{(2n)!}\right)\psi_0(x,y,z,t). \quad (71)$$

The solution is

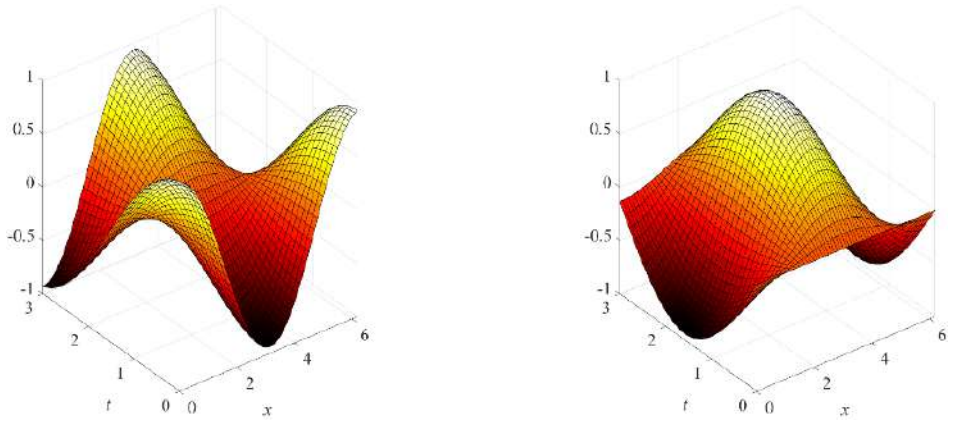
$$\psi(x,y,z,t) = e^{-it}\cos x \cos y \cos z\ . \quad (72)$$

t	x	Exact solution	LVIM solution		Absolute error	
		$\mathrm{Re}[\psi_e]$	$\mathrm{Re}[\psi_{7(66)}]$	$\mathrm{Re}[\psi_{7(71)}]$	$\lvert\mathrm{Re}[\psi_e - \psi_{7(66)}]\rvert$	$\lvert\mathrm{Re}[\psi_e - \psi_{7(71)}]\rvert$
0.1	0.5	8.1975×10^{-1}	8.1975×10^{-1}	8.1975×10^{-1}	0	0
0.2	1.0	4.9712×10^{-1}	4.9712×10^{-1}	4.9712×10^{-1}	0	0
0.3	1.5	5.8278×10^{-2}	5.8278×10^{-2}	5.8278×10^{-2}	0	0
0.4	2.0	-2.1108×10^{-1}	-2.1107×10^{-1}	-2.1108×10^{-1}	9.5825×10^{-6}	0
0.5	2.5	-2.7253×10^{-1}	-2.7245×10^{-1}	-2.7253×10^{-1}	7.8937×10^{-5}	0
0.6	3.0	-6.5743×10^{-2}	-6.5167×10^{-2}	-6.5743×10^{-2}	5.7624×10^{-4}	0
0.7	4.0	2.5536×10^{-1}	2.5909×10^{-1}	2.5544×10^{-1}	3.7281×10^{-3}	7.6174×10^{-5}
0.8	4.5	1.1646×10^{-1}	1.1901×10^{-1}	1.1716×10^{-1}	2.5537×10^{-3}	6.9973×10^{-4}
0.9	5.0	-2.1334×10^{-1}	-2.2275×10^{-1}	-2.0828×10^{-1}	9.4101×10^{-3}	5.0649×10^{-3}
1.0	6.0	-8.9238×10^{-1}	-1.0253	-7.8051×10^{-1}	1.3296×10^{-1}	1.1187×10^{-1}

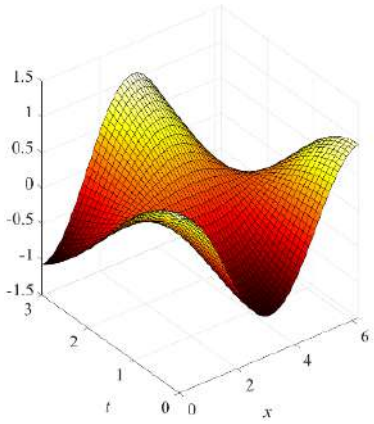
Table 3. Comparison of the real parts of exact solution ψ_e in (68), approximate solutions $\psi_{7(66)}$, $\psi_{7(71)}$ in (66), (71) for $n = 7$ and absolute errors at $y, z = 0.25$ in Example 2.

t	x	Exact solution	LVIM solution		Absolute error	
		$\mathrm{Im}[\psi_e]$	$\mathrm{Im}[\psi_{7(66)}]$	$\mathrm{Im}[\psi_{7(71)}]$	$\lvert\mathrm{Im}[\psi_e - \psi_{7(66)}]\rvert$	$\lvert\mathrm{Im}[\psi_e - \psi_{7(71)}]\rvert$
0.1	0.5	-8.2249×10^{-2}	-8.2249×10^{-2}	-8.2249×10^{-2}	0	0
0.2	1.0	-1.0077×10^{-1}	-1.0077×10^{-1}	-1.0077×10^{-1}	0	0
0.3	1.5	-3.1837×10^{-2}	-3.1837×10^{-2}	-3.1837×10^{-2}	0	0
0.4	2.0	3.2874×10^{-1}	3.2874×10^{-1}	3.2874×10^{-1}	0	0
0.5	2.5	7.0099×10^{-1}	7.0098×10^{-1}	7.0099×10^{-1}	1.0555×10^{-5}	0
0.6	3.0	9.2707×10^{-1}	9.2697×10^{-1}	9.2707×10^{-1}	9.6475×10^{-5}	0
0.7	4.0	5.5798×10^{-1}	5.5714×10^{-1}	5.5814×10^{-1}	8.3511×10^{-4}	1.6644×10^{-4}
0.8	4.5	1.6000×10^{-1}	1.5937×10^{-1}	1.6096×10^{-1}	6.3028×10^{-4}	9.6130×10^{-4}
0.9	5.0	-1.5937×10^{-1}	-1.5673×10^{-1}	-1.5559×10^{-1}	2.6465×10^{-3}	3.7836×10^{-3}
1.0	6.0	-1.2721×10^{-1}	-8.2092×10^{-2}	-1.1126×10^{-1}	4.5114×10^{-2}	1.5946×10^{-2}

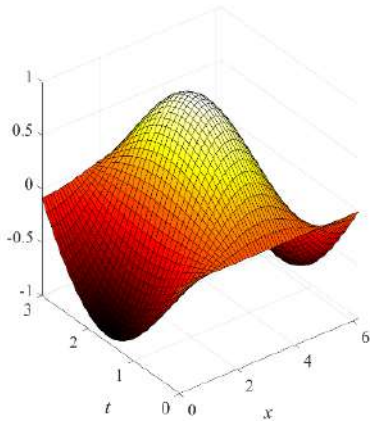
Table 4. Comparison of the imaginary parts of exact solution ψ_e in (68), approximate solutions $\psi_{7(66)}$, $\psi_{7(71)}$ in (66), (71) for $n = 7$ and absolute errors at $y, z = 0.25$ in Example 2.



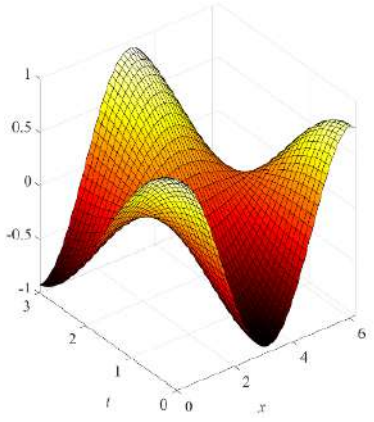
(a) $\mathrm{Re}[\psi_e]$ **(b)** $\mathrm{Im}[\psi_e]$



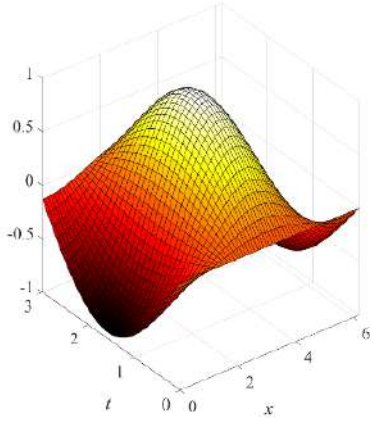
(c) $\mathrm{Re}[\psi_{7(66)}]$



(d) $\mathrm{Im}[\psi_{7(66)}]$

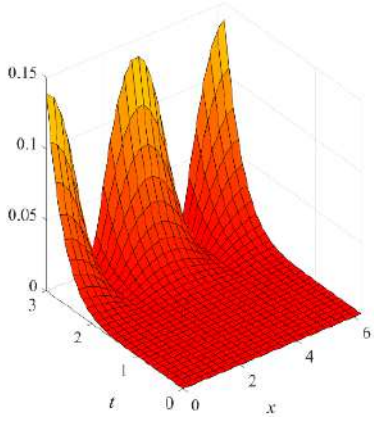


(e) $\mathrm{Re}[\psi_{7(71)}]$

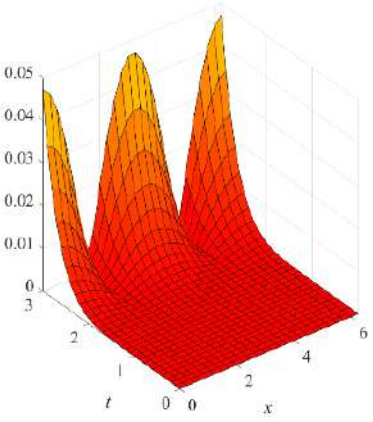


(f) $\mathrm{Im}[\psi_{7(71)}]$

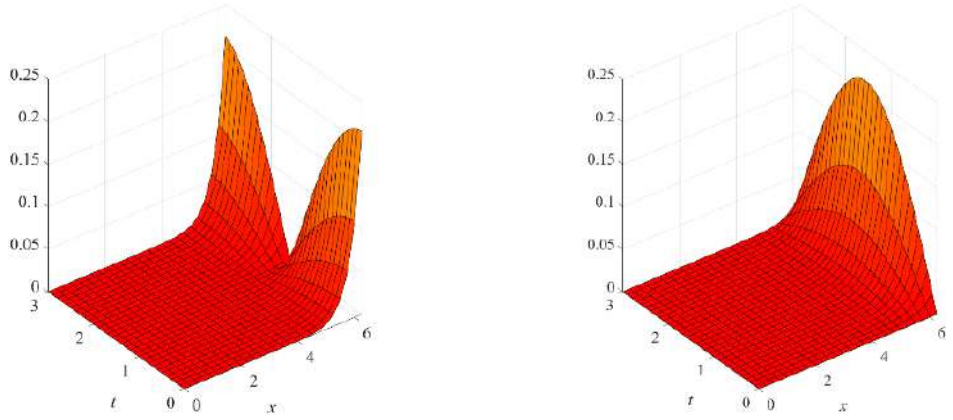
Figure 3. The graphs of the real and imaginary parts of exact solution ψ_e in (68) and approximate solutions $\psi_{7(66)}$, $\psi_{7(71)}$ in (66), (71) for $n = 7$ at $y, z = 0.25$ in Example 2.



(a) $\left|\mathrm{Re}[\psi_e - \psi_{7(66)}]\right|$



(b) $\left|\mathrm{Im}[\psi_e - \psi_{7(66)}]\right|$



(c) $\left|\mathrm{Re}\left[\psi_e - \psi_{7(71)}\right]\right|$ **(d)** $\left|\mathrm{Im}\left[\psi_e - \psi_{7(71)}\right]\right|$

Figure 4. The graphs of the real and imaginary parts absolute errors of exact solution ψ_e in (68) and approximate solutions $\psi_{7(66)}$, $\psi_{7(71)}$ in (66), (71) for $n = 7$ at $y, z = 0.25$ in Example 2.

6 Conclusion

In the present research, three dimensional linear and nonlinear time dependent Schrödinger equations are investigated by Laplace variational iteration method. Lagrange multipliers are easily determined by the Laplace transform. Different iteration formulas are formed for Laplace variational iteration method. In the example problems of linear and nonlinear time-dependent Schrödinger equations, approximate solutions that rapidly converge to the exact solution have been obtained by using two different iteration formulas of the method. It has been shown that, depending on the choice of iteration formula used, approximate solutions that converge to the exact solution faster can be obtained.

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Study on Conformable Fractional Differenial Systems of Second Order with Nonlocal-Delay Conditions in Banach Spaces

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Abstract: In this work, we study the existence and uniqueness of the mild solutions, for the problem of conformable fractional differential system of second order, with nonlocal-delay conditions in Banach space. First, we give the form of the mild solution by using Cosine and Sine functions of operators and conformable fractional calculus. Furthermore, existence and sufficient conditions of uniqueness are given by Krasnoseleskii fixed point and Banach contraction Theorems. In the end, we give an illustrative example of mains results.

Contents

1. Introduction

2. Preliminaries

3. Main results

4. Example

5. Conclusion

6. References

Keywords: Conformable fractional derivative, Nonlocal Cauchy problem, Finite delay.

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1. Introduction

Fractional differential equations have recently been proved to be valuable tools in the modeling of many phenomena in various fields of science and engineering [1, 2, 6, 8, 13, 14, 18]. there are many different definitions of fractional operators such caputo and Riemann-Liouville fractional derivatives and integral.

A new fractional operator labeled conformable derivatives is introduced by khalil on 2014 [15], which verifies almost all the properties of classical derivative such as linearity property, quotient, product and chain rules contrary to the other fractional definitions and furthermore it is also possible for a non-differentiable function to have a conformable derivative and hence fractional differential equations of non-differentiable functions are easily solvable numerically. "Nonlocal conditions" concept has been introduced by L. Byszewski [9, 10], for the extension of problems based on classical conditions. For Caputo fractional derivative, a fractional Cauchy problem of order $\beta \in (1, 2)$ with nonlocal condition was treated by KavithaWilliams and al [17]. In recent years many authors did works on conformable derivative such as Abdeljawad and all who extended and giving some new concepts and proprieties on the Co-FD [16], and others [5, 11]. But, there are a few researchers used conformable derivative to study fractional differential equations with time-delay [4, 12].

In this paper, we will discuss the nonlocal fractional differential equation with time delay in second order in the frame of the conformable derivative of order $0 < \alpha \leq 1$ in Banach space, by

using Cosine and Sine functions of operators and fixed point theorems. Consider the nonlocal-delay fractional differential system of second order has the following form:

$$\begin{cases} T_\alpha^2(x)(t) = Ax(t) + f(t, x(t)), \quad 0 < t \le a, \quad 0 < \alpha < 1 \\ x_0(\tau) + \left(g\left(x_{t_1}, x_{t_2}, \ldots, x_{t_n}\right)\right)(\tau) = \phi(\tau), \tau \in [-l, 0], \quad T_\alpha(x)(0) = x_1 \end{cases} \tag{0.1}$$

x(⋅) takes values in the Banach space X ; Tα(x)(t) is the conformable fractional derivative of order $0 < \alpha \le 1$. A is the infinitesimal generator of a strongly continuous cosine family $\{C(t)\}t\geq 0$ in a Banach space X, f : [0, a] × X → X satisfying some assumptions and x0, x1 are elements of space X. The history function x_t is defined by $x_t(\rho) = x(t + \rho)$, $\rho \in [-l, a]$.

Let l > 0, C = C ([−l, 0],X) be the space of continuous functions from [−l, 0] → X endowed by the norme sup; defined by$\| y \|* = Sup_{\rho\epsilon[-l,0]}|y(\rho)|$, For every y ∈ C.

For $0 < t_1 < t_2 <. < t_n \le a$; $g : C^n \rightarrow C$ is an appropriate function.

2. Preliminaries

Recalling some preliminary facts on the new conformable fractional calculus.

Definition 1 .[15] The conformable derivative of x of order $0 < \alpha < 1$ at t > 0 is defined as :

$$T_\alpha(x)(t) = \lim_{\varepsilon \to 0} \frac{x\left(t + \varepsilon t^{1-\alpha}\right) - x(t)}{\varepsilon}$$

When the limit exists, we say that x is (α)-differentiable at t.

for t = 0, $T_\alpha(x)(0) = \lim_{t \to 0^+} T_\alpha(x)(t)$

The fractional integral Iα(.) associated with the conformable fractional derivative defined by:

$I^\alpha(x)(t) = \int_0^t s^{\alpha-1} x(s) \mathrm{d}s$

Definition 2 [16]. The Laplace transform of a function x(.) is defined by

$$\mathcal{L}(x(t))(\lambda) := \int_0^{+\infty} e^{-\lambda t} x(t) \mathrm{d}t, \quad \lambda > 0$$

The adapted transform is given by the following definition:

The fractional Laplace transform of order $\alpha \in]0, 1]$ of a function x(.) is defined by

$$\mathcal{L}_\alpha(x(t))(\lambda) := \int_0^{+\infty} t^{\alpha-1} e^{-\lambda\left(t^\alpha/\alpha\right)} x(t) \mathrm{d}t, \quad \lambda > 0$$

Proposition 1 [16] If x(.) is a differentiable function, then we have the following results:

$$I^\alpha\left(T_\alpha(x)(.)\right)(t) = x(t) - x(0)$$

$$\mathcal{L}_\alpha\left(T_\alpha(x)(t)\right)(\lambda) = \lambda \mathcal{L}_\alpha(x(t))(\lambda) - x(0)$$

Remark : For two functions x(.) and y(.), we have

$$\mathcal{L}_\alpha\left(x\left(\frac{t^\alpha}{\alpha}\right)\right)(\lambda) = \mathcal{L}(x(t))(\lambda) \tag{2.1}$$

$$\mathcal{L}_\alpha\left(\int_0^t s^{\alpha-1}x\left(\frac{t^\alpha - s^\alpha}{\alpha}\right)y(s)\mathrm{d}s\right)(\lambda) = \mathcal{L}(x(t))(\lambda)\mathcal{L}_\alpha(y(t))(\lambda) \tag{2.2}$$

Definition 3 [3]. A one-parameter family $(C(t))_{t\in\mathbb{R}}$ of bounded linear operators on X is called a strongly continuous cosine family if and only if:

1. C(0) = I;

2. C(s + t) + C(s − t) = 2C(s)C(t) for all t, s ∈ R;

3. t → C(t)x is continuous for each fixed x ∈ X.

We define the sine family by

$$S(t)x := \int_0^t C(s)x\,ds$$

The infinitesimal generator A of a strongly continuous cosine family $(C(t))_{t\in\mathbb{R}}, (S(t))_{t\in\mathbb{R}}$ on X is defined by: D(A) = {x ∈ X, t → C(t)x is a twice continuously differentiable function}; $Ax = \frac{d^2C(0)x}{dt^2}$

Proposition 2 [3] The following assertions are true.

1. There exist constants K ≥ 1 and ω ≥ 0 such that

$$|S(t) - S(s)| \le K\left|\int_s^t \exp(\omega|r|)\right|dr \quad \text{for all } t,s \in \mathbb{R} \tag{2.3}$$

2. If x ∈ X and t, s ∈ R, then: $\int_s^t S(r)x\,dr \in D(A)$ and $A\int_s^t S(r)x\,dr = C(t)x - C(s)x$

3. If t → C(t)x is differentiable, then: $S(t)x \in D(A)$ and $\frac{dC(t)}{dt}x = AS(t)x$

4. For λ such that Re(λ) > ω, we have: $\lambda^2 \in \rho(A)$, (ρ(A) is the resolvent set of A)

$$\lambda\left(\lambda^2 I - A\right)^{-1}x = \int_0^{+\infty} e^{-\lambda t}C(t)x\,dt \quad \text{and} \quad \left(\lambda^2 I - A\right)^{-1}x = \int_0^{+\infty} e^{-\lambda t}S(t)x\,dt \quad x \in X$$

Lemma 1 [20] (Krasnoselskii fixed point theorem). Let M be a bounded closed convex subset of Banach space X. Assume that P1, P2 be mappings of M into X such that P1x+P2y ∈ M for every pair x, y ∈ M. If P1 is a contraction and P2 is completely continuous. Then the operator equation P1x + P2x = x has a solution on M.

3 Main results

Using the fractional Laplace transform in equation (1.1), we get:

$$\mathcal{L}_\alpha(x(t))(\lambda) = \lambda\left(\lambda^2 - A\right)^{-1}\left[\phi(0) + \left(g\left(x_{t_1}, x_{t_2}, \ldots, x_{t_n}\right)\right)(0)\right] + \left(\lambda^2 - A\right)^{-1}x_1 + \left(\lambda^2 - A\right)^{-1}\mathcal{L}_\alpha(f(t, x(t)))(\lambda)$$

According to the inverse fractional Laplace transform, we find:

$$x(t)=C\left(\frac{t^{\alpha}}{\alpha}\right)\left[\phi(0)+\left(g\left(x_{t_1},x_{t_2},\ldots,x_{t_n}\right)\right)(0)\right]+S\left(\frac{t^{\alpha}}{\alpha}\right)x_1+\int_0^t r^{\alpha-1}S\left(\frac{t^{\alpha}-r^{\alpha}}{\alpha}\right)f\ (r,x_r)dr$$

Thus, we can introduce the following definition.

Definition 4 We say that x ∈ C is a mild solution of equation (1.1) if the following assertion is true:

$$\begin{cases} x(t)=C\left(\frac{t^{\alpha}}{\alpha}\right)\left[\phi(0)-\left(g\left(x_{t_1},x_{t_2},\ldots,x_{t_n}\right)\right)(0)\right]+S\left(\frac{t^{\alpha}}{\alpha}\right)x_1 \\ +\int_0^t r^{\alpha-1}S\left(\frac{t^{\alpha}-r^{\alpha}}{\alpha}\right)f\ (r,x_r)dr; \quad t\in[0,a] \\ \\ x_0(\tau)+\left(g\left(x_{t_1},x_{t_2},\ldots,x_{t_n}\right)\right)(\tau)=\phi(\tau); \quad \tau\in[-l,0] \end{cases} \tag{3.1}$$

we introduce the following assumptions for proving the main results :

(H1): The operator {S(t)} is compact for t ≥ 0.

(H2): The function f(t, ⋅) : X → X is continuous, and for all x ∈ X, f(⋅ , x) : [0, a] → X is strongly measurable.

(H3): for all γ > 0, there exists a function $\psi_{\gamma}\in L^{\infty}\left([0,a],\mathbb{R}^{+}\right)$ such that:

$$\sup \| f\ (t,x)\| \le \psi_{\gamma}(t) \text{ for all t} \in [0, a]$$

(H4): There exists K > 0 such that:

$$\left\| g\ x_{t_1},x_{t_2},\ldots,x_{t_n} - g\ y_{t_1},y_{t_2},\ldots,y_{t_n} \right\| \le K \| x-y \|, for x,y \in C([-l,a],\mathcal{X})$$

$$\left\| g\ x_{t_1},x_{t_2},\ldots,x_{t_n} - g\ y_{t_1},y_{t_2},\ldots,y_{t_n} \right\| \le K \| x-y \| \text{ , for x, y} \in C([-l, a],X);$$

Theorem 1 Assume that the hypotheses (H1) − (H4) are satisfied, then the fractional nonlocal-delay differential system (1) has a mild solution if x (0) ∈ D(A) and

$$K \sup_{t\in[0,a]}\left|C\left(\frac{t^{\alpha}}{\alpha}\right)\right|<1 \tag{3.2}$$

Proof 1 Choosing

$$\gamma \ge \frac{\sup_{t\in[0,a]}\left|C\left(\frac{t^{\alpha}}{\alpha}\right)\right|\left[\|\phi\|_{\infty}+\|(g(0,0,\ldots,0))\|\right]+\sup_{t\in[0,a]}\left|S\left(\frac{t^{\alpha}}{\alpha}\right)\right|\left[\frac{a^{\alpha}}{\alpha}\left|\psi_{\gamma}\right|_{L^{\infty}([0,a],\mathbb{R}^{+})}+\|x_1\|\right]}{\left(1-K\sup_{t\in[0,a]}\left|C\left(\frac{t^{\alpha}}{\alpha}\right)\right|\right)} \tag{3.3}$$

Let Bγ = {x ∈ C, |x| ≤ γ}. Next, for x ∈ Bγ define the operators P1 and P2 by:

$$\begin{cases} \mathcal{P}_1(x)(t) = C\left(\frac{t^\alpha}{\alpha}\right)\left[\phi(0) - \ g\ \ x_{t_1},,x_{t_n}\quad (0)\right] + S\left(\frac{t^\alpha}{\alpha}\right)x_1, t \in [0,a] \\ \mathcal{P}_1 x\ \ (\tau) = \phi(\tau) - \ g\ \ x_{t_1}, x_{t_2},\ldots,x_{t_n}\quad (\tau)); \quad \tau \in [-l,0] \end{cases}$$

and

$$\begin{cases} \mathcal{P}_2(x)(t) = \int_0^t r^{\alpha-1} S\left(\frac{t^\alpha - r^\alpha}{\alpha}\right) f(r, x_r) dr, \quad t \in [0,a], \quad t \in [0,a] \\ \mathcal{P}_2 x\ \ (\tau) = 0, \qquad\qquad \tau \in [-l,0] \end{cases}$$

Step 1: By using assumptions (H1) −(H4) and (3.3) , we show that $\mathcal{P}_1$ (x) + $\mathcal{P}_2$ (y) ∈ Bγ whenever x, y ∈ Bγ. Moreover, the operator $\mathcal{P}_1$ is a contraction on Bγ.

Step 2: To prove: $\mathcal{P}_1$ is a contraction on Bγ. For any x, y ∈ Bγ and t ∈ [0, a], we have

$$\begin{aligned} \| \ \mathcal{P}_1 x\ \ (t) - \ \mathcal{P}_1 y\ \ (t)\| &= \left\| C\left(\frac{t^\alpha}{\alpha}\right)\left[\ \ g\ \ x_{t_1}, x_{t_2},\ldots,x_{t_n}\quad (0) - \ g\ \ y_{t_1}, y_{t_2},\ldots,y_{t_n}\quad (0)\right]\right\| \\ &\le \sup_{t\in[0,a]}\left|C\left(\frac{t^\alpha}{\alpha}\right)\right| \left\| \ g\ \ x_{t_1}, x_{t_2},\ldots,x_{t_n}\quad - \ g\ \ y_{t_1}, y_{t_2},\ldots,y_{t_n}\quad \right\| \\ &\le \sup_{t\in[0,a]}\left|C\left(\frac{t^\alpha}{\alpha}\right)\right| K \ \| \ x - y \ \|, \end{aligned}$$

for τ ∈ [−l, 0], we have: $\| \ \mathcal{P}_1 x\ \ (\tau) - \ \mathcal{P}_1 y\ \ (\tau)\| \le K \ \| \ x - y \ \| \le \sup_{t\in[0,a]}\left|C\left(\frac{t^\alpha}{\alpha}\right)\right| K \ \| \ x - y \ \|$.

By using (3.2), we conclude that $\mathcal{P}_1$ is a contraction.

Step 3: To prove that $\mathcal{P}_2$ is a completely continuous operator.

Claim 1: $\mathcal{P}_2$ is continuous. Indeed

Let $x_n \ \subset B_\gamma$ such that $x_n \to x$ in Bγ. Then, by using assumptions (H2)- (H3), we obtain that:

$f\ \ r, x_n(r)\ \to f(r, x(r))$ as n → +∞, and

$$\left\| r^{\alpha-1}\left[f\ \ r, x_n(r)\ \ - f(r, x(r))\right]\right\| \le 2\psi_\gamma(r) r^{\alpha-1}$$

On the other hand, we have:

$$\left|\mathcal{P}_2\ \ x_n\ \ - \mathcal{P}_2(x)\right| \le \sup_{t\in[0,a]}\left|S\left(\frac{t^\alpha}{\alpha}\right)\right| \int_0^a r^{\alpha-1}\left\| f\ \ r, x_n(r)\ \ - f(r, x_r)\right\| dr$$

By using the Lebesgue dominated convergence theorem, we get:

$$\lim_{n\to+\infty}\left|\mathcal{P}_2\ \ x_n\ \ - \mathcal{P}_2(x)\right| = 0$$

Claim 2: We prove that $\mathcal{P}_2 x, x \in B_\gamma$ is equicontinuous at t ∈ [0, a]. Let $t_1, t_2 \in]0,\tau]$ such that $t_1 < t_2$ .We have; $\lim_{n\to+\infty} \left|\mathcal{P}_2 x\ t_2 - \mathcal{P}_2 x(t_1)\right| = 0$. Indeed

$$\mathcal{P}_2(x)\ t_2 - \mathcal{P}_2(x)\ t_1 = \int_0^{t_1} r^{\alpha-1}\left[S\left(\frac{t_2^\alpha - r^\alpha}{\alpha}\right) - S\left(\frac{t_1^\alpha - r^\alpha}{\alpha}\right)\right] f(s,x(s))ds$$

$$+\int_{t_1}^{t_2} r^{\alpha-1} S\left(\frac{t_2^\alpha - r^\alpha}{\alpha}\right) f(r,x_r)dr$$

Therefore, by using (4) we obtain:

$$\left\|\mathcal{P}_2(x)\ t_2 - \mathcal{P}_2(x)\ t_1\right\| \le \left|\psi_\gamma\right|_{L^\infty\ [0,\tau],\mathbb{R}^+} \left[\frac{K}{\omega^2}\left(\exp\left(\frac{\omega t_2^\alpha}{\alpha}\right) - \exp\left(\frac{\omega t_1^\alpha}{\alpha}\right)\right) + \sup_{t\in[0,\tau]}\left|S\left(\frac{t^\alpha}{\alpha}\right)\right|\left(\frac{t_2^\alpha - t_1^\alpha}{\alpha}\right)\right]$$

$$\le \left|\psi_\gamma\right|_{L^\infty\ [0,a],\mathbb{R}^+} \left(\frac{K}{\omega\alpha} + \sup_{t\in[0,a]}\left|S\left(\frac{t^\alpha}{\alpha}\right)\right|\right)\left[\frac{t_2^\alpha - t_1^\alpha}{\alpha}\right].$$

We conclude that the functions $\mathcal{P}_2 x, x \in B_\gamma$ is equicontinuous at t ∈ [0, a].

Claim 3 : We prove that $\mathcal{P}_2 x, x \in B_\gamma$ is relatively compact in X.

For some fixed t ∈]0,a[, let ε ∈]0,t [, $x \in B_\gamma$ and define the operator $\mathcal{P}_2^\varepsilon$ by:

$$\mathcal{P}_2^\varepsilon(x)(t) = \int_0^{t^\alpha - \varepsilon^\alpha\ ^{\frac{1}{\alpha}}} r^{\alpha-1} S\left(\frac{t^\alpha - r^\alpha}{\alpha}\right) f(r,x_r)dr$$

The relative compactness of $\mathcal{P}_2^\varepsilon(x)(t), x \in B_\gamma$ in X is guaranteed by the compactness of (S(t))t>0 according assumption (H1). and we have:

$$\left\|\mathcal{P}_2^\varepsilon(x)(t) - \mathcal{P}_2(x)(t)\right\| \le \left|\psi_\gamma\right|_{L^\infty\ [0,a],\mathbb{R}^+} \sup_{t\in[0,\tau]}\left|S\left(\frac{t^\alpha}{\alpha}\right)\right| \frac{\varepsilon^\alpha}{\alpha}$$

Therefore, we conclude that {P2(x)(t), x ∈ Bγ} is relatively compact in X. It's clear that the set $\mathcal{P}_2^\varepsilon(x)(t), x \in B_\gamma$ is compact. And $\mathcal{P}_2 x, x \in B_\gamma$ is relatively compact in X for all t ∈ [0, a]

Finally, By using Arzela–Ascoli theorem, we prove that $\mathcal{P}_2$ is relatively compact, and the Krasnoselskii fixed point theorem completes the proof.

To obtain the uniqueness of the mild solution, we will need the following assumption:

(H5) : There exists a constant L > 0 such that $\| f(t,y) - f(t,x) \| \le L \| y - x \|$ for all x, y ∈ X and t ∈ [0, a]

Theorem 2 Assume that (H2) − (H5) hold. Then the Fractional nonlocal-delay differential system (1.1) has a unique mild solution provided that: x(0) ∈ D(A) and

$$K \sup_{t\in[0,a]}\left|C\left(\frac{t^{\alpha}}{\alpha}\right)\right| + \left(L\frac{a^{\alpha}}{\alpha}\right)\sup_{t\in[0,a]}\left|S\left(\frac{t^{\alpha}}{\alpha}\right)\right| < 1 \tag{3.4}$$

.

Proof 2 Let t ∈ [0, a] and define the operator $\mathcal{P} : \mathcal{C} \to \mathcal{C}$ by:

$$\begin{cases} \mathcal{P}(x)(t) = C\left(\frac{t^{\alpha}}{\alpha}\right)\left[\phi(0) + \ g\ \ x_{t_1}, x_{t_2}, \ldots, x_{t_n} \quad (0)\right] + S\left(\frac{t^{\alpha}}{\alpha}\right)x_1 \\ +\int_0^t r^{\alpha-1} S\left(\frac{t^{\alpha}-r^{\alpha}}{\alpha}\right) f(r, x_r)dr \quad , t \in [0, a] \\ \mathcal{P}(x)(\tau) = \phi(\tau) - \ g\ \ x_{t_1}, x_{t_2}, \ldots, x_{t_n} \quad (\tau) \quad , \tau \in [-l, 0] \end{cases}$$

Next, let be x, y ∈ C, then we have: for t ∈ [0, a]

$$\begin{aligned} \mathcal{P}(y)(t) - \mathcal{P}(x)(t) = C\left(\frac{t^{\alpha}}{\alpha}\right)[\ g\ \ y_{t_1}, y_{t_2}, \ldots, y_{t_n} \quad (0) - \ g\ \ x_{t_1}, x_{t_2}, \ldots, x_{t_n} \quad (0)] \\ +\int_0^t r^{\alpha-1} S\left(\frac{t^{\alpha}-s^{\alpha}}{\alpha}\right)[f(r, y_r) - f(r, x_r)]dr \end{aligned}$$

by using (H2)−(H5) , we obtain:

$$\left|\mathcal{P}(y)(t) - \mathcal{P}(x)(t)\right| \le \left[K \sup_{t\in[0,a]}\left|C\left(\frac{t^{\alpha}}{\alpha}\right)\right| + \left(L\frac{a^{\alpha}}{\alpha}\right)\sup_{t\in[0,a]}\left|S\left(\frac{t^{\alpha}}{\alpha}\right)\right|\right] | \ y - x \ |$$

Then we get

$$\left\|\mathcal{P}y - \mathcal{P}x\right\| \le \left[K \sup_{t\in[0,a]}\left|C\left(\frac{t^{\alpha}}{\alpha}\right)\right| + \left(L\frac{a^{\alpha}}{\alpha}\right)\sup_{t\in[0,a]}\left|S\left(\frac{t^{\alpha}}{\alpha}\right)\right|\right]\left\|y - x\right\|$$

For $\tau \in [-l, 0]$, we have:

$$\begin{aligned} \left|\mathcal{P}(y)(\tau) - \mathcal{P}(x)(\tau)\right| &= \left|C\left(\frac{t^{\alpha}}{\alpha}\right)[\ g\ \ y_{t_1}, y_{t_2}, \ldots, y_{t_n} \quad (\tau) - \ g\ \ x_{t_1}, x_{t_2}, \ldots, x_{t_n} \quad (\tau)]\right| \le K \sup_{t\in[0,a]}\left|C\left(\frac{t^{\alpha}}{\alpha}\right)\right| | \ y - x \ | \\ &\le K \sup_{t\in[0,a]}\left|C\left(\frac{t^{\alpha}}{\alpha}\right)\right| + \left(L\frac{a^{\alpha}}{\alpha}\right)\sup_{t\in[0,a]}\left|S\left(\frac{t^{\alpha}}{\alpha}\right)\right| | \ y - x \ | \end{aligned}$$

Therefore, using (3.4) and applying Banach contraction theorem, P has a unique fixed point in C. and the problem (1.1) has a unique solution.

4 Example

To illustrate our main results, we present the following example: Consider the nonlocal conformable fractional partial differential system with delay of the form:

$$\begin{cases} \dfrac{\partial^{\frac{1}{2}}}{\partial t^{\frac{1}{2}}}\dfrac{\partial^{\frac{1}{2}}u(t,x)}{\partial t^{\frac{1}{2}}} = \dfrac{\partial^2 u(t,x)}{\partial x^2} + \dfrac{\mid u(t,x)\mid}{1+\mid u(t,x)\mid} + \displaystyle\int_0^t \dfrac{\mid u(s,x)\mid}{1+\mid u(s,x)\mid}ds, \quad (t,x)\in]0,1]\times]0,\pi[\\ u(\tau,x) + \displaystyle\sum_{i=0}^{n} c_i u_{t_i}(\tau,x) = (\phi(\tau))(x), \quad \tau\in[-l,0] \end{cases} \tag{4.1}$$

with the following nonlocal conditions:

$$u(t,0) = u(t,\pi) = 0 \quad \text{and} \qquad \frac{\partial^{\frac{1}{2}}u(0,x)}{\partial t^{\frac{1}{2}}} = x_1 \in [0,\pi] \tag{4.2}$$

where $0 < t_1 < \cdots < t_n < 1$ and $c_1,\ldots,c_n$ are given real constants such that:

$$\sum_{i=1}^{n} \mid c_i \mid \; < \; \frac{4}{10}$$

We take: $\alpha = \dfrac{1}{2}$ and $T_\alpha = \dfrac{\partial^{\frac{1}{2}}}{\partial t^{\frac{1}{2}}}$

Let $X = L^2([0,\pi])$ and define the operator A: X → X by:

$$A = \frac{\partial^2(\cdot)}{\partial x^2} \quad \text{and} \quad D(A) = \; v\in H^2(0,\pi), v(0) = v(\pi) = 0$$

The operator A generates a cosine family $(C(t))_{t\in\mathbb{R}}, (S(t))_{t\in\mathbb{R}}$. Moreover, we have |C(t)| ≤ 1and |S(t)| ≤ 1 for all t ∈ [0, 1]. Next, we consider the following transformations:

$$\xi(t)(x) = u(t,x), \quad f(t,\xi(t)) = \frac{\mid \xi(t)\mid}{1+\mid \xi(t)\mid} + \int_0^t \frac{\mid \xi(s)\mid}{1+\mid \xi(s)\mid}ds$$

$$g \;\; x_{t_1}, x_{t_2}, \ldots, x_{t_n} \qquad (\tau) = \sum_{i=1}^{n} c_i \xi_{t_i} \quad \tau$$

Then the system (4.1) with the conditions (4.2) can be written in the form:

$$\begin{cases} T_{\frac{1}{2}}^2(\xi)(t) = A\xi(t) + f(t,\xi(t)), \quad 0 < t \le a, \quad 0 < \alpha < 1 \\ \xi_0^2(\tau) + \;\; g \;\; \xi_{t_1}, \xi_{t_2}, \ldots, \xi_{t_n} \quad (\tau) = \phi(\tau), \tau\in[-l,0], \quad T_{\frac{1}{2}}(\xi)(0) = \xi_1 \end{cases}$$

So, we can verify all the hypotheses of theorem 2. Therefore, the above Cauchy problem has a unique mild solution.

5. Conclusion

This paper uses conformable derivative to study the problem of fractional differential equation of second order, with nonlocal and time delay conditions in Banach space, and obtains the results of the existence and the sufficient conditions for the uniqueness of the mild solution.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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Physics Informed Neural Network Method for the Numerical Solution of Benney-Roskes/Zakharov-Rubenchik System

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1. Abstract:

Differential equations are utilized to model numerous natural phenomena in computational science [1]. For the majority of scenarios, numerical methods are the only means available to solve these equations with discretization [2].

In literature, various numerical methods such as the finite element method [3], finite difference method [4], spectral methods [5], meshless have been used to solve various kinds of PDEs.

Consequently, artificial neural networks are progressively employed to construct continuous solution functions for solving various types of differential equations.

In this study, a method known as the Physics Informed Neural Network (PINN) is suggested as a numerical solution for the Benney-Roskes and Zakharov-Rubenchik system.

Numerical experiments show that the present PINN method provides accurate solutions on the considered computational space-time domain.

2. Introduction:

In this paper we present the numerical solution of the Benney-Roskes/Zakharov-Rubenchik (BR/ZR) system via the PINN method under certain conditions.

The BR system was introduced by Benney and Roskes in 1969 in the context of gravity waves [6]. Zakharov and Rubenchik [7] introduced the ZR system, which describes the interaction of a spectrally narrow high-frequency wave packet of small amplitude with a low-frequency oscillation of acoustic type.

Saut and Ponce [8] reduced the BR/ZR system to a nonlinear Schrodinger equation with nonlocal terms and derivatives of the unknown, and studied the local well-posedness of the

Cauchy problem. Luong et al. revisit the Cauchy problem for the two- and three-dimensional Zakharov-Rubenchik system and analyse its perturbation by a line soliton [9]. Quintero and Cordero worked on the nonlinear orbital instability of ground state standing waves for a Benney-Roskes/Zakharov-Rubenchik system in two and three spatial directions [10]. In [11] Martinez and Palacios studied the decay properties for the solutions to the initial value problem of the 1-D ZR/BR system. In [12] Gonul and Ozemir study the Lie symmetry algebra and present exact solutions of the Benney-Roskes/Zakharov-Rubenchik (BR/ZR) system

$$\begin{aligned} &i\psi_t + \frac{\epsilon}{2}\omega_{22}\psi_{xx} + \frac{\epsilon}{2}\omega_{11}\psi_{zz} = \epsilon|\psi|^2\psi + \frac{\epsilon}{2g\omega}(g^2k^2 - \omega^4)\rho\psi + \epsilon k\varphi_z\psi, \\ &\rho_t - c_g\rho_z + h(\varphi_{xx} + \varphi_{zz}) + \frac{2gk}{\omega}(|\psi|^2)_z = 0, \\ &\varphi_t - c_g\varphi_z + g\rho + \frac{1}{\omega^2}(g^2k^2 - \omega^4)|\psi|^2 = 0. \end{aligned} \tag{2.1}$$

Gonul and Ozemir showed that (1.1) is identical to system

$$i\psi_t + a\psi_{xx} + \varepsilon_0\psi_{yy} = (b - \omega m)|\psi|^2\psi - \omega\psi\left\{\mathrm{m}\frac{\partial}{\partial t} + (\mathrm{mc} - \mathrm{d})\frac{\partial}{\partial y}\right\}\varphi,$$

$$\varphi_{xx} + (1 - mc^2)\varphi_{yy} - m\varphi_{tt} - 2mc\varphi_{yt} = \left\{\mathrm{m}\frac{\partial}{\partial t} + (\mathrm{mc} - \mathrm{d})\frac{\partial}{\partial y}\right\}|\psi|^2. \tag{2.2}$$

If the following transformations applied

$$\Psi = e^{i\theta}\psi(v), \quad \varphi = \phi(v), \quad v = v_1x + v_2y + v_0t, \quad \theta = \theta_1x + \theta_2y + \theta_0t$$

where v_i, θ_i, $i = 0,1,2$ are real constants, (2.2) reduces the BR/ZR system to the system of nonlinear ordinary differential equations

$$\begin{aligned} &(av_1^2 + \epsilon_0v_2^2)\Psi'' + i(2av_1\theta_1 + 2\epsilon_0v_2\theta_2 + v_0)\Psi' - (a\theta_1^2 + \epsilon_0\theta_2^2 + \theta_0)\Psi \\ &\qquad + (mw - b)\Psi^3 + w(mv_0 + mcv_2 - dv_2)\Psi\Phi' = 0, \\ &2(dv_2 - mv_0 - mcv_2)\Psi\Psi' + (v_1^2 + v_2^2 - m(v_0 + cv_2)^2)\Phi'' = 0 \end{aligned} \tag{2.3}$$

which requires

$$2av_1\theta_1 + 2\varepsilon_0v_1\theta_1 + v_0 = 0.$$

If the values of parameters are taken as equal to 1 except $w = 3$ and $\theta_2 = -3/2$, the exact solution of (2.3) BR/ZR system can be obtained as

$$\psi(\mathrm{v}) = \sqrt{17}\,\mathrm{sech}\left(\sqrt{\frac{17}{8}}(x + y)\right), \ \Phi(\mathrm{v}) = -\sqrt{34}\tanh\left(\sqrt{\frac{17}{8}}(x + y)\right). \tag{2.4}$$

In the literature, the exact solution of the (2 + 1) dimensional BR/ZR system was found by taking some values [12] and we focused on finding numerical solutions with the PINN method.

3.Physics Informed Neural Network (PINN):

Physics-informed neural networks (PINNs) are a deep learning technique used in solving partial differential equations (PDEs) [13]. It allows computers to compute the partial derivative of function accurately and rapidly. We take time parameter t as a special component of x, and **Ω** contains the temporal domain. The initial condition can be simply treated as a special type of Dirichlet boundary condition on the spatio-temporal domain.

PINNs generate approximate solutions to PDEs by training a neural network to minimize a loss function consisting of terms representing the misfit of the initial and boundary conditions along the boundary of the space-time domain as well as the PDE residual at selected point in the interior [15]. It provides taking derivative of a function directly.

By considering z = [x, t]^T , PINN method assumes the following approximate function

$$u\theta(z) := W^{L}\,\sigma^{L}\,(W^{L-1}\ \sigma^{L-1}\,(\ldots\sigma^{1}\,(W^{0}z + b^{0})) + b^{L-1}) + b^{L} \tag{3.1}$$

θ consists weight matrices (W) and bias vectors (b),
Weight matrices; $W^{0} \in R^{mx2}$, $W^{i} \in R^{mxm}$ (i = 1, 2, ..., L−1) and $W^{L} \in R^{1xm}$ denotes the number of neurons, L is the number of layers.
Bias vectors; $b^{i} \in R^{mx1}$ (i = 1, 2, ..., L−1) and $b^{L} \in R$,
σ(.) function is the activation function that transforms the neural network into a nonlinear form.
The approximation in (3.1) is an approximation function of the multilayer feed forward neural network. Here m is the number of neurons in the layers and L is the number of layers. Equation (3.1) contains vectorized parameters θ = W^{0}, b^{0}, W^{1}, b^{1}, ..., W^{L}, b^{L} and it is necessary to optimize this approximation function to provide the given PDE.

$$R_\theta(x,t) := \theta_t u_\theta(x,t) - L[u_\theta](x,t)+q(x,t) \tag{3.2}$$

is continuous residual function. The L operator in this function is defined as,

$$L : V \rightarrow R;\ V = u(x, t) : R^2 \rightarrow R \tag{3.3}$$

$$L[u_\theta](x,t)=\frac{1}{\Gamma(-\alpha)}\lim_{N_X\to\infty}\left(\frac{1}{h_X}\right)\sum_{k=0}^{N_X}\frac{\Gamma(k-\alpha)}{\Gamma(k+1)}u_\theta(x-(k-1)h,t) \tag{3.4}$$

The PINN method minimizes the residual function on the collocation points defined on the fractional differential equation calculation region, while the function tries to optimize the θ parameter so that the boundary and initial conditions are met. Let's take the collocation points as

$$X_i^r = \{(x_i^r,\ \ t_i^r)\}_{i=1}^{N_r} C[x_L, x_R].[0,T] \tag{3.5}$$

Although these collocation points are random in the classical PINN method, we take them as equally spaced points. Since the derivative approximation of the equation in (2.3) requires x_i points with step size h.

The PINN method is obtained by minimizing the following loss functional for the solution of equation

$$\varphi_\theta := \varphi_\theta^r(X^r) + \varphi_\theta^b(X_b^r) + \varphi_\theta^s(X_s^r) \tag{3.6}$$

There are three different loss functions in equation (8) and they belong to the residue, initial condition and boundary conditions, respectively. We chose to optimize all loss functions using Adam method. Residual loss function can be written as

$$\varphi_\theta^r(X^r) = \frac{1}{N_r}\sum_{i=1}^{N_r}|r_\theta(x_i^r, t_i^r)|^2 \tag{3.7}$$

On the other hand, functions of initial and boundary conditions are defined as

$$\varphi_\theta^b(X_b^r) = \frac{1}{N_b}\sum_{i=1}^{N_b}\left|u_\theta\left(x_i^b, 0\right) - g(x_i^b)\right|^2 \tag{3.8}$$

$$\varphi_\theta^s(X_s^r) = \frac{1}{N_s}\sum_{i=1}^{N_s}|u_\theta(x_i^s, t_i^s) - u_b(x_i^s, t_i^s)|^2 \tag{3.9}$$

$$u_b(x_i^s, t_i^s) = \begin{cases} 0 \; if \; x_i^s = x_L \\ 1 \; if \; x_i^s = x_R \end{cases} \tag{3.10}$$

Thus, the solution of the following optimization problem
Θ^*= A = arg min_φ $\theta(x^R)$

Residue; returns to the optimum $\varphi*$ parameters that minimize the initial and boundary functions.

In this study, the codes are written on the Python library Tensor Flow and the optimal $\varphi*$ parameters were obtained with the Adam algorithm

5.Numerical Experiments and Results

We consider (1.3) BR/ZR system with the parameters taking equal to 1 except $w = 3 \; and \;\; \theta_2 = -3/2$. We apply the proposed PINN method for various layers and iterations and get the errors listed in Table 1, and sketch the graph of the errors to show the optimization of the solution in figure 2. Also we give the graphs of the solutions $\psi(v)$ and $\Phi(v)$ obtained in figure 1.

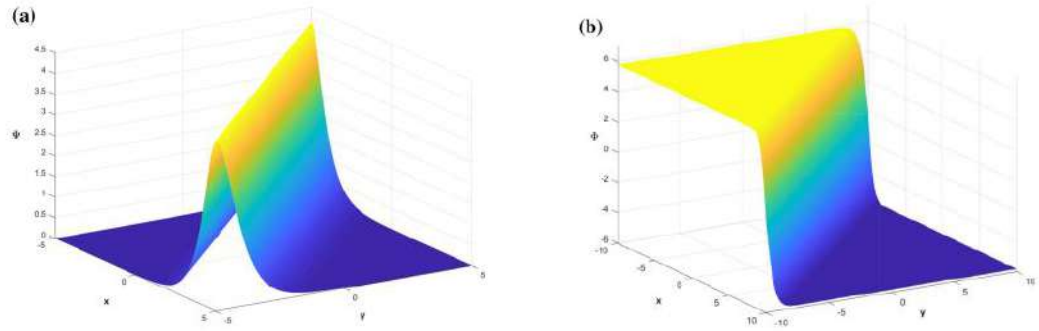

Figure 1. Plots of (1.4) **(a)** $\psi(v)$ and $(\boldsymbol{b})$ $\Phi(v)$ for t=0.

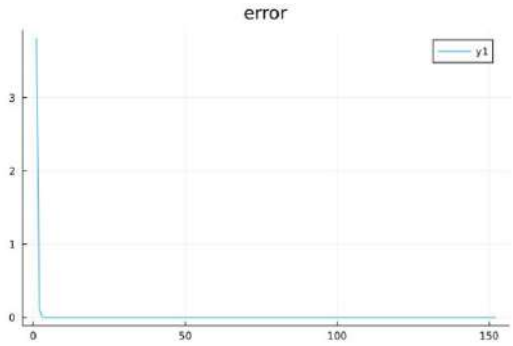

Figure 2. Error graph.

N	Iteration	PINN
15	**1000**	**3.86E-09**
15	**2000**	**5.76E-04**
50	**1000**	**1.55E-10**
50	**2000**	**2.81E-10**

Tablo 1: Error analysis of PINN in different parameters.

Conclusions:

In this article, we focused on the numerical solution of the Benney-Roskes/Zakharov-Rubenchik system of equations using the PINN method. The neural networks modelled for the solutions of the problems were obtained by using the Adam algorithm in the Tensor Flow library.

With the help of numerical experiments, it has been shown that the PINN method can be an alternative convergent approach method. The effect of the number of the collocation points,

and the number of neurons in each layer on the absolute error are presented with quantitative observations.

Keywords: Benney-Roskes and Zakharov-Rubenchik System, PINN Method.

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On New Banach Sequence Spaces $e_0^{a,b}(u,p,q)$ and $e_c^{a,b}(u,p,q)$ and Their Duals

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Abstract: In this study, we define the Banach sequence spaces $e_0^{a,b}(u,p,q)$ and $e_c^{a,b}(u,p,q)$ resulting from the generalized (p,q)-Euler matrix $E^{a,b}(p,q)$. We give some topological properties and inclusion relations of these newly created spaces. We also identified $\alpha-,\beta-$and $\gamma-$duals of the spaces.

Keywords: Sequence spaces, BK-space**,** (p,q) calculus, $\alpha-,\beta-$and $\gamma-$duals

1. Introduction and Preliminaries

Chakrabarti and Jagannathan [1] introduced the (p,q)-number to provide a generalization for various types of q-oscillator algebras. (p,q)-calculus is widely used in various mathematical areas such as operator theory, approximation theory, algebra, special functions, combinatorics and integral equations. Several researchers have employed (p,q)-theory in various branches of mathematics to expand the scope of single-parameter q-calculus theory ([1-8]). Jagannathan and Rao [2] are defined the (p,q)-number $(0<q<p\leq 1)$ as

$$[k]_{p,q}=\frac{p^k-q^k}{p-q}$$

for any number k. When $p=1$ it is clear that $[k]_{p,q}$ receives its version q.

Corcino [3] defined the (p,q)-coefficient as

$$\begin{bmatrix}n\\k\end{bmatrix}_{p,q}=\begin{cases}\dfrac{[n]_{p,q}!}{[n-k]_{p,q}!\,[k]_{p,q}!}, & (n\geq k),\\ 0, & (k>n).\end{cases}$$

Where $[k]_{p,q}!$ is the (p,q)-factorial of k such that $[k]_{p,q}!=\ [k]_{p,q}[k-1]_{p,q}\dots[2]_{p,q}[1]_{p,q}$.

Duran et al. [8] defined the (p,q)-binomial formula as

$$(a+b)_{p,q}^n=\begin{cases}(a+b)(qa+qb)(q^2a+p^2b)\dots(q^{n-1}a+p^{n-1}b), & n\geq 1,\\ 1, & n=0,\end{cases}$$

$$=\sum_{k=0}^{n}\begin{bmatrix}n\\k\end{bmatrix}_{p,q}p^{\binom{n-k}{2}}q^{\binom{k}{2}}a^kb^{n-k}.$$

When $p=1$ it is clear that (p,q)-binomial formula receives its version q-binomial formula.

Let us denote the space of all real or complex sequence with w. We write the sequence spaces of all convergent, null, bounded and absolutely p −summable sequences by c, c_0, l_∞ and l_p respectively. Also we will denote the space of all bounded, convergent and absolutely convergent series with bs, cs, and l_1 respectively.

Let $\Theta = (\theta_{nk})$ be an infinite matrix of real or complex numbers and X, Y be subsets of w. We write $\Theta_n(x) = \sum_k \theta_{nk} x_k$ for $n, k \in \mathbb{N}$. For a sequence space X, the matrix domain of an infinite matrix Θ is defined by

$$X_\Theta = \{x = (x_k) \in w : \Theta x \in X\}$$

which is also a sequence space. We denote with (X, Y) the class of all matrices A such that $\Theta : X \to Y$. Recently, the literature focused on the creation of new sequence spaces through the matrix domain, the investigation of their algebraic and topological properties and the study of matrix transformations has expanded. To gain a better understanding of the theory of sequence spaces, you can refer to these valuable articles ([9]-[14]).

Altay and Başar [9] defined Euler sequence space e_0^r and e_∞^r. They also calculated $\alpha-, \beta-$and $\gamma-$ and continuous duals of these spaces. Let $E^a = (e_{nk}^a)$ denotes the Euler matrix of order a and $0 < a < 1$, $E^a = (e_{nk}^a)$ is defined by

$$e_{nk}^a = \begin{cases} \binom{n}{k}(1-a)^{n-k}a^k, & 0 \le k \le n, \\ 0, & k > n, \end{cases}$$

for all $k, n \in \mathbb{N}$. Let $0 < a < 1$ and $a + b \neq 1$, the binomial matrix is defined by

$$e_{nk}^{a,b} = \begin{cases} \frac{1}{(a+b)^n}\binom{n}{k}a^n b^{n-k}, & 0 \le k \le n, \\ 0, & k > n. \end{cases}$$

When $a + b = 1$ it is clear that $E^{a,b}$ reduces to E^a.

Bekar [15] is defined q-Cesàro matrix $C^q = (c_{nk}^q)$ such that for $0 < q < 1$,

$$c_{nk}^q = \begin{cases} \frac{q^k}{[n+1]_q}, & 0 \le k \le n, \\ 0, & k > n. \end{cases}$$

This matrix serves as a mathematical tool for the development of sequences. New research is being conducted using post-quantum (or (p, q)-) versions of these matrices to construct sequence spaces. In particular, there have been studies on creating sequence spaces using the quantum analogs of specific matrices. For example, Demiriz and Şahin [16] focused on q-

Cesàro sequence spaces X_0^q and X_c^q, generated by the q-Cesàro matrix in c_0 and c respectively. As a natural extension of these investigations, Yaying et al. [14] have expanded its domains X_k^q and X_∞^q in l_k and l_∞ respectively and explored associated operator ideals. In more recent times, Yaying et al. [17] have introduced (p,q)-Euler sequence spaces $e_k^{a,b}(p,q)$ and $e_k^{a,b}(p,q)$ and have examined certain geometric properties of these spaces. Additionally, Yaying et al. [18] have introduced (p,q)-Euler sequence spaces, $e_0^{a,b}(p,q)$ and $e_0^{a,b}(p,q)$. Furthermore, Yaying et al. [19] have introduced q-Euler sequence apaces $e_0^{a,b}(q)$ and $e_0^{a,b}(q)$.

Yaying et al. [17] defined the generalized (p,q)-Euler matrix $E^{a,b}(p,q) = \left(e_{nk}^{a,b}\right)$ of order (a,b) as

$$\left(e_{nk}^{a,b}\right) = \begin{cases} \frac{1}{(a+b)^n} \begin{bmatrix} n \\ k \end{bmatrix}_{p,q} p^{\binom{n-k}{2}} q^{\binom{k}{2}} a^k b^{n-k}, & 0 \le k \le n, \\ 0, & k > n, \end{cases}$$

where a and b nonnegative real numbers and $0 < q < p \le 1$.

We observe that $\lim_{n\to\infty} e_{nk}^{a,b} = 0$ and $\sum_{k=0}^{n} e_{nk}^{a,b} = 1$ $(n \in \mathbb{N}_0)$. Consequently, $E^{a,b}(p,q)$ is regular. Additionally we can refer to, $E^{a,b}(p,q)$ as the (p,q) analog of the binomial matrix $E^{a,b}$. The definition provided is justifiable, as one can see that when $p = 1$ and q approaches 1, the matrix $E^{a,b}(p,q)$ reduces to the binomial matrix $E^{a,b}$. It is worth noting that when $p = 1$, $E^{a,b}(p,q)$ transforms into the triangular matrix $E^{a,b}(q)$, with elements in the form of $\begin{bmatrix} n \\ k \end{bmatrix}_q q^{\binom{k}{2}} a^k b^{n-k}/(a+b)_q^n$. We refer to $E^{a,b}(q)$ as the $q-$analog of the binomial matrix $E^{a,b}$. Furthermore, when $b = 1 - a$, the matrix $E^{a,b}(p,q)$ transforms into the triangular matrix $E^a(p,q)$ with elements in the form of $(1/(a + (1 - a))_{p,q}^n) \begin{bmatrix} n \\ k \end{bmatrix}_{p,q} p^{\binom{n-k}{2}} q^{\binom{k}{2}} a^k (1 - a)^{n-k}$.

2. Main Results

In this section, we first introduced the Banach sequence spaces $e_0^{a,b}(u,p,q)$ and $e_c^{a,b}(u,p,q)$ resulting from the generalized (p,q)-Euler matrix $E^{a,b}(p,q)$. We give some topological properties and inclusion relations of these newly created spaces.

Definition 2.1. Let a, b nonnegative real numbers, $0 < q < p \le 1$ and $u = (u_k)$ be the bounded sequence. Then, the generalized (p,q)-Euler spaces $e_0^{a,b}(u,p,q)$ and $e_c^{a,b}(u,p,q)$ are defined by

$$e_0^{a,b}(u,p,q)=\{x=(x_k)\in w:\ \lim_{n\to\infty}\frac{1}{(a\ +\ b)_{p,q}^{n}}\sum_{k=0}^{n}\begin{bmatrix}n\\k\end{bmatrix}_{p,q}u_k p^{\binom{n-k}{2}}q^{\binom{k}{2}}a^k b^{n-k}x_k=0\},$$

$$e_c^{a,b}(u,p,q)=\{x=(x_k)\in w:\ \lim_{n\to\infty}\frac{1}{(a\ +\ b)_{p,q}^{n}}\sum_{k=0}^{n}\begin{bmatrix}n\\k\end{bmatrix}_{p,q}u_k p^{\binom{n-k}{2}}q^{\binom{k}{2}}a^k b^{n-k}x_k \text{ exists}\}.$$

The sapaces $e_0^{a,b}(u,p,q)$ and $e_c^{a,b}(u,p,q)$ transform into the specific spaces mentioned below, contingent upon the selection of (u,p,q) and (a,b):

i) If $p=1$and $(u_k)=(1,1,\dots)$, $e_0^{a,b}(u,p,q)$ and $e_c^{a,b}(u,p,q)$ reduce to q-binomial spaces $e_0^{a,b}(q)$ and $e_c^{a,b}(q)$, respectively, due to Yaying et al. [19].

ii) If $p=1$, $(u_k)=(1,1,\dots)$ and $q\to 1^-$, $e_0^{a,b}(u,p,q)$ and $e_c^{a,b}(u,p,q)$ reduce to binomial spaces $e_0^{a,b}$ and $e_c^{a,b}$, respectively, due to Bişgin [20].

iii) If $b=1-a$ and $(u_k)=(1,1,\dots)$, $e_0^{a,b}(u,p,q)$ and $e_c^{a,b}(u,p,q)$ reduce to Euler spaces $e_0^{a}(p,q)$ *and* $e_c^{a}(p,q)$, respectively.

Let us define a sequence $z=(z_n)$ in terms of sequence $x=(x_k)$ by

$$z_n=(E^{a,b}(p,q)x)_n=\frac{1}{(a+b)_{p,q}^{n}}\sum_{k=0}^{n}\begin{bmatrix}n\\k\end{bmatrix}_{p,q}p^{\binom{n-k}{2}}q^{\binom{k}{2}}a^k b^{n-k}x_k \qquad (1)$$

for every $n\in\mathbb{N}_0$. The sequence z is called $E^{a,b}(p,q)$-transform of the sequence x. Due to (1), we write

$$x_n=\sum_{k=0}^{n}(-1)^{n-k}\frac{\begin{bmatrix}n\\k\end{bmatrix}_{p,q}p^{\binom{n-k}{2}}b^{(n-k)}(a+b)_{p,q}^{k}}{a^n q^{\binom{n}{2}}} \qquad (2)$$

for every $n\in\mathbb{N}_0$.

Theorem 2. 1. The sequence spaces $e_0^{a,b}(u,p,q)$ and $e_c^{a,b}(u,p,q)$ are BK-spaces with the following norm,

$$\|x\|_{e_0^{a,b}(u,p,q)}=\|x\|_{e_c^{a,b}(u,p,q)}=\sup_{n\in\mathbb{N}_0}\left|\frac{1}{(a+b)_{p,q}^{n}}\sum_{k=0}^{n}\begin{bmatrix}n\\k\end{bmatrix}_{p,q}u_k p^{\binom{n-k}{2}}q^{\binom{k}{2}}a^k b^{n-k}x_k\right|. \qquad (3)$$

Proof. Omited.

Theorem 2. 2. The sequence spaces $e_0^{a,b}(u,p,q)$ and $e_c^{a,b}(u,p,q)$ are linearly isomorphic to c_0 and c, respectively, that is, $e_0^{a,b}(u,p,q)\cong c_0$ and $e_c^{a,b}(u,p,q)\cong c$.

Proof. We'll just do the proof for $e_0^{a,b}(u,p,q) \cong c_0$ for the other it can be done similarly. Define the mapping $T: e_0^{a,b}(u,p,q) \to c_0$ by $Tx = E^{a,b}(p,q)x$ for all $x \epsilon e_0^{a,b}(u,p,q)$. $T$is linear and one to one. Let $z = (z_n) \in c_0$ and $x = (x_n)$ be as defined in (3). Then we have

$$\lim_{n\to\infty} \frac{1}{(a+b)_{p,q}^n} \sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix}_{p,q} u_k p^{\binom{n-k}{2}} q^{\binom{k}{2}} a^k b^{n-k} x_k$$

$$= \lim_{n\to\infty} \frac{1}{(a+b)_{p,q}^n} \sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix}_{p,q} u_k p^{\binom{n-k}{2}} q^{\binom{k}{2}} a^k b^{n-k} \left(\sum_{r=0}^{k} (-1)^{k-r} \frac{\begin{bmatrix} k \\ r \end{bmatrix}_{p,q} q^{\binom{k-r}{2}} t^{(k-r)} (a+b)_{p,q}^r}{a^k q^{\binom{k}{2}}} \right)$$

$$= \lim_{n\to\infty} z_n = 0$$

Thus, $x \in e_0^{a,b}(u,p,q)$ and $T: e_0^{a,b}(u,p,q) \to c_0$. This is what we wanted to prove.

Theorem 2. 3. The following inclusions strictly hold:

(i) $e_0^{a,b}(u,p,q) \subset e_c^{a,b}(u,p,q)$,

(ii) $c_0 \subset e_0^{a,b}(u,p,q)$ and $c \subset e_c^{a,b}(u,p,q)$.

Proof. (i) It is obvious.

(ii) We know that the matrix $E^{a,b}(p,q)$ is regular, therefore $c_0 \subset e_0^{a,b}(u,p,q)$ holds. Now let us consider the sequence $x = (1,-1,1,-1,\ldots)$; then, $E^{a,b}(p,q)$-transform of the sequence x, that is, $E^{a,b}(p,q) = ((-a+b^n)/(a+b)_{p,q}^n))$ $(n \in \mathbb{N}_0)$, is a sequence the space c_0, although x is not sequence in c_0. This proves that the inclusion strictly holds. The details of the latter case are omitted as the proof follows a similar line of reasoning as discussed above.

3. $\alpha-, \beta-$ and $\gamma-$Duals

In this section, we calculate the $\alpha-, \beta-$and $\gamma-$duals of $e_0^{a,b}(u,p,q)$ and $e_c^{a,b}(u,p,q)$.

Definition 3.1. The $\alpha-, \beta-$and $\gamma-$duals of $X \subset w$ are given by

$$X^\alpha = \{\zeta = (\zeta_n) \in w: \zeta x = (\zeta_n x_n) \in l_1 \ \forall x \in X\},$$

$$X^\beta = \{\zeta = (\zeta_n) \in w: \zeta x = (\zeta_n x_n) \in c_s \ \forall x \in X\},$$

$$X^\gamma = \{\zeta = (\zeta_n) \in w: \zeta x = (\zeta_n x_n) \in b_s \ \forall x \in X\},$$

respectively.

Here, we introduce established lemmas attributed to Stielglitz and Tietz [21], pivotal in substantiating our theorems. Throughout the remainder of the paper, $\mathcal{N}$ denotes the set encompassing all finite subsets of $\mathbb{N}_0$.

Lemma 3.1 Let $\Theta = (\theta_{nk})$ be an infinite matrix, then $\Theta \in (c_0, l_1)$ if and only if

$$\sup_{N\in\mathcal{N}}\left(\sum_{k=0}^{\infty}\left|\sum_{n\in N}\theta_{nk}\right|\right) < \infty. \tag{4}$$

Lemma 3.2. Let $\Theta = (\theta_{nk})$ be an infinite matrix, then

(i) $\Theta \in (c_0, l_\infty)$ if and only if

$$\sup_{n\in\mathbb{N}_0}\left(\sum_{k=0}^{n}|\theta_{nk}|\right) < \infty, \tag{5}$$

(ii) $\Theta \in (c_0, c)$ if and only if (5) holds with

$$\lim_{n\to\infty}\theta_{nk} \text{ exists for every } k \in \mathbb{N}_0.$$

Theorem 3.1. Let $w_k = \frac{1}{|u_k|}$, define the set D_1 by

$$D_1 = \left\{\zeta = (\zeta_n) \in w:\ sup_{N\in\mathcal{N}}\sum_{k=0}^{\infty}\left|\sum_{n\in N}(-1)^{n-k}\frac{\begin{bmatrix}n\\k\end{bmatrix}_{p,q} q^{\binom{n-k}{2}} b^{(n-k)}(a+b)^k_{p,q}}{a^n q^{\binom{n}{2}}}\zeta_n\right| w_k\right\}$$

Then, $[e_0^{a,b}(u,p,q)]^\alpha = [e_c^{a,b}(u,p,q)]^\alpha = D_1$.

Proof. Let $(\zeta_n) \in w$, from equality (2), we have

$$\zeta_n x_n = \sum_{k=0}^{\infty}(-1)^{n-k}\frac{\begin{bmatrix}n\\k\end{bmatrix}_{p,q} q^{\binom{n-k}{2}} b^{(n-k)}(a+b)^k_{p,q}}{a^n q^{\binom{n}{2}}}\zeta_n z_k \tag{6}$$

$$= (\Phi^{a,b}(p,q)z)_n, (n \in \mathbb{N}_0),$$

where $\Phi^{a,b}(p,q) = (\phi_{nk}^{a,b}(p,q))$ is triangle defined by

$$\phi_{nk}^{a,b}(p,q) = \begin{cases}(-1)^{n-k}\dfrac{\begin{bmatrix}n\\k\end{bmatrix}_{p,q} q^{\binom{n-k}{2}} b^{(n-k)}(a+b)^k_{p,q}}{a^n q^{\binom{n}{2}}}\zeta_n, & 0 \le k \le n,\\ 0, & k > n.\end{cases}$$

Thus, we observe on using (6) that $\zeta x = (\zeta_n x_n) \in l_1$ whenever $x \in e_0^{a,b}(u,p,q)$ if and only if $\Phi^{a,b}(p,q)z \in l_1$ whenever $z \in c_0$. Thus we realize that $\zeta = (\zeta_n) \in [e_0^{a,b}(u,p,q)]^\alpha$ iff $E^{a,b}(p,q) \in (c_0, l_1)$. Thus, from Lemma 1, we conclude that $[e_0^{a,b}(u,p,q)]^\alpha = D_1$.

Theorem 3.2. Let $w_k = \frac{1}{|u_k|}$, consider the sets D_i, $2 \leq i \leq 4$, defined by,

$$D_2 = \left\{ \zeta = (\zeta_n) \in w: \ \sum_{r=k}^{\infty} (-1)^{r-k} \frac{\begin{bmatrix} r \\ k \end{bmatrix}_{p,q} q^{\binom{r-k}{2}} b^{(r-k)} (a+b)_{p,q}^k}{a^r q^{\binom{r}{2}}} \zeta_r \, w_k \text{ exists } \forall k \in \mathbb{N}_0 \right\},$$

$$D_3 = \left\{ \zeta = (\zeta_n) \in w: \ \sup_{N \in \mathcal{N}} \sum_{k=0}^{n} \left| \sum_{r=k}^{n} (-1)^{r-k} \frac{\begin{bmatrix} r \\ k \end{bmatrix}_{p,q} q^{\binom{r-k}{2}} b^{(r-k)} (a+b)_{p,q}^k}{a^r q^{\binom{r}{2}}} \zeta_r \right| w_k < \infty \right\},$$

$$D_4 = \left\{ \zeta = (\zeta_n) \in w: \ \lim_{n \to \infty} \sum_{k=0}^{n} \sum_{r=k}^{n} (-1)^{r-k} \frac{\begin{bmatrix} r \\ k \end{bmatrix}_{p,q} q^{\binom{r-k}{2}} b^{(r-k)} (a+b)_{p,q}^k}{a^r q^{\binom{r}{2}}} \zeta_r \, w_k \text{ exists} \right\}.$$

Then, $[e_0^{a,b}(u,p,q)]^\beta = D_2 \cap D_3$ and $[e_c^{a,b}(u,p,q)]^\beta = D_2 \cap D_3 \cap D_4$.

Proof. Let $(\zeta_n) \in w$ and $z = (z_n)$ be the $E^{a,b}(p,q)$- transform of the sequence $x = (x_n)$, from equality (2), we have

$$\sum_{k=0}^{n} \zeta_k x_k = \sum_{k=0}^{n} \left\{ \sum_{r=0}^{k} (-1)^{r-k} \frac{\begin{bmatrix} r \\ k \end{bmatrix}_{p,q} q^{\binom{r-k}{2}} b^{(r-k)} (a+b)_{p,q}^k}{a^r q^{\binom{r}{2}}} z_r \right\} \zeta_k$$

$$= \sum_{k=0}^{n} \left\{ \sum_{r=0}^{k} (-1)^{r-k} \frac{\begin{bmatrix} r \\ k \end{bmatrix}_{p,q} q^{\binom{r-k}{2}} b^{(r-k)} (a+b)_{p,q}^k}{a^r q^{\binom{r}{2}}} \zeta_r \right\} z_k$$

$$= (\Psi^{a,b}(p,q)z)_n, (n \in \mathbb{N}_0),$$

where $\Psi^{a,b}(p,q) = (\psi_{nk}^{a,b}(p,q))$ is given by

$$\psi_{nk}^{a,b}(p,q) = \begin{cases} (-1)^{r-k} \dfrac{\begin{bmatrix} r \\ k \end{bmatrix}_{p,q} q^{\binom{r-k}{2}} b^{(r-k)} (a+b)_{p,q}^k}{a^r q^{\binom{r}{2}}} \zeta_n, & 0 \leq k \leq n, \\ 0, & k > n. \end{cases}$$

Thus, we observe on using (6) that $\zeta x = (\zeta_n x_n) \in c_s$ whenever $x \in e_0^{a,b}(u,p,q)$ if and only if $\Psi^{a,b}(p,q)z \in c$ whenever $z \in c_0$. Thus, $\zeta = (\zeta_n) \in [e_0^{a,b}(u,p,q)]^\beta$ iff $\Psi^{a,b}(p,q) \in (c_0, c)$. Thus, from Lemma 3.2.(ii), we conclude that $[e_0^{a,b}(u,p,q)]^\beta = D_2 \cap D_3$.

Theorem 3.3. Let $w_k = \frac{1}{|u_k|}$. Then, $[e_0^{a,b}(u,p,q)]^\gamma = [e_c^{a,b}(u,p,q)]^\gamma = D_3$.

Proof. The proof follows a pattern akin to the preceding theorem, with the distinction that Lemma 3.2.(i) is employed instead of Lemma 3.2.(ii).

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Cutting Parameter Optimization based on Online Temperature Measurements

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Abstract: The deformation of metallic materials during the machining operation requires a significant amount of energy. During the chip formation process and due to the plastic deformation of the metal and the friction along the tool-part interface, the thermal loads generated are strongly impacted by the cutting factors. Thus, the choice of optimized cutting conditions is essential to control the quality of the work required. The aim of the present experimental study is to optimize the cutting parameters using temperature measurements. The average temperature of the cutting tool is studied using a FLIR A325sc type infrared camera. Optimal cutting parameters for each performance metric were obtained using the Taguchi techniques.

Keywords: machining, cutting conditions, optimization, thermography, temperature measurement

I. INTRODUCTION

The choice of cutting conditions is a very important factor in machining. With the increasingly rapid development of machine tools and the use of new high-performance materials for the tool-part couple, cutting temperature is an essential concern in machining. To be highly competitive, the objectives sought by the mechanical manufacturing industries are to produce high quality products with low cost and time constraints. To achieve them, one of the solutions is to optimize machining parameters such as cutting speed, depth of cut and feed. The effects of cutting parameters (cutting speed, depth of cut, feed per revolution,) on surface roughness, tool wear, cutting forces related to turning operations have been studied in depth. Authors in [1] used the Response Surface Method (RSM) with a factorial design to predict surface roughness when machining high strength materials. Authors in [2] studied the machinability of hard steel in turning by the response surface method. Authors in [3] used a multiplicative model to predict surface roughness and tool vibration. The context of the current work is based on the use of full factorial type experimental designs based on the experimental characterization of the temperature measurement for the optimization of the cutting conditions during the turning operation. Many techniques have been developed during the recent years that address this type of problem. Authors in [4] propose a general classification of the different techniques for modeling and optimizing input-output relationships in metal cutting processes. They are classified into two methods, classical optimization techniques and unconventional techniques. The current work is oriented on the study of the heat flow in the zone of the cut represented by the measurement of the temperature.

Various techniques of temperature measurement have been studied and evaluated [5- 8]. The optical infrared method has been used in [9-10].

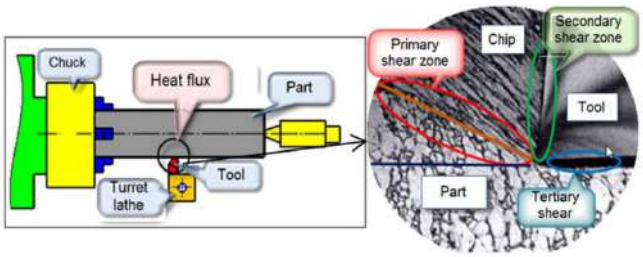

Fig. 1. Sources of heat generation in machining.

At the heart of the process, two main phenomena are in interaction:

- A very strong plastic deformation in the shear zones.
- The friction of the chip on the rake face of the tool.

One may also add the friction of the clearance face on the newly machined surface. The energy generated is then largely transformed into a significant heat flux which modifies and deteriorates the machined surface and the cutting tool shown in Figure 1.

In this paper, an experimental measurement methodology is proposed and the results are presented using an acquisition instrumentation, equipped with a single infrared camera with advantages such as contactless measurement, measurement without or with very little disturbance between the surface of the studied object and its surrounding environment, the capacity for real-time measurement, wide range of operating temperatures, the ability to adapt to any type of material, and a unit processing of the measured values. The correlation between the cutting conditions and the temperature appears on the flowchart represented in Figure 2.

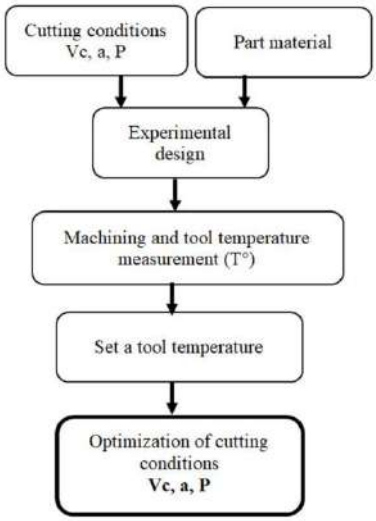

Fig. 2. Correlation between cutting conditions and temperature.

In industry and in order to make the best choices, very often one empirical method (Taylor's law, Kronenberg model, etc.) is used to calculate and optimize the cutting parameters before machining. Knowing that the low cumulative gains generate significant profits, we propose an approach that allows refining the manufacturing conditions after following the experimental tests.

II. MATERIALS AND METHODS

The infrared camera used is the Flir A325sc, coupled with the FLIR Research software. The inserts are made of tungsten carbide with a beak radius r = 0.4mm (Figure 3). The specimens are made of S185 steel, a material widely used in industry. For the machining we used a conventional parallel lathe made by TOS TRENCIN type (Figure 4).



Fig. 3. Thermal camera FLIR A325sc and tool holder with the used insert.



Fig. 4. Machining station with tool, specimen, and camera

The conducted experimental tests and the calibration of the camera are widely discussed in [11]. Before using the results, the data are retrieved from the camera software (FLIR Quick Plot) for viewing the machining sequences. Figure 5 represents the acquisition chain.



Fig. 5. Chain of acquisition and processing of thermal data.

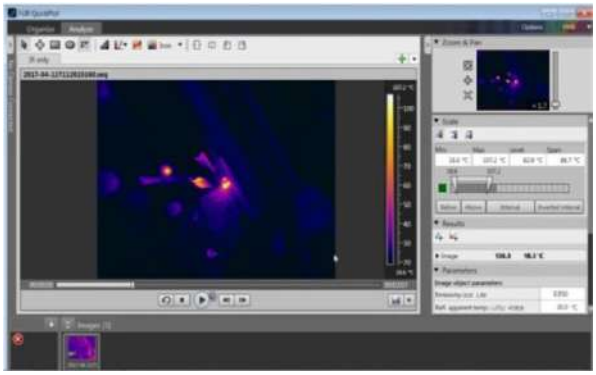

Fig. 6. Software interface.

The recovery of the temperatures is made after tracing the curve represented by Figure 7. The curve is divided into three zones:

- Zone 1 represents the machining initiation.
- Zone 2 determines the stabilization part of the section, from the latter we recover the temperatures for analysis.
- Zone 3 represents the tool release and the end of the turning operation.



Fig. 7. Software interface and curve areas studied.

III. TEMPERATURE PREDICTION MODEL

The relationship between the inputs which are the machining parameters (cutting speed, feed, depth of cut) as independent variables vary according to an N^3 type design of experiments. In a 4^3type experiment, each parameter has 4 levels, which gives us 64 experiments (full plan). We carried out 64 machining operations, each time using a new cutting edge. The expanded linear model of the type second-order equation is:

$$Y= b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_{11}X_1^2 + b_{22}X_2^2 + b_{33}X_3^2 + b_{12}X_1X_2 + b_{13}X_1X_3 + b_{23}X_2X_3$$

where Y = T is the estimated response, X_1, X_2, X_3 respectively represent the control factors (cutting speed, feed, and depth of cut), and b_0, ..., b_{23} are the coefficients of the related factors. Constants and exponents are determined by the least squares method. After careful reading of the sequences recorded by the camera, we determined the temperature of the tip of the tool according to the cutting parameters as shown in Table I

TABLE I. CUTTING PARAMETERS

Parameter	Code	Levels			
		1	2	3	4
Cutting speed Vc (m/min)	X_1	78	112	157	220
Feed rate a (mm/rev)	X_2	0.08	0.11	0.14	0.20
Depth of cut P (mm)	X_3	0.25	0.5	0.75	1
Turning conditions	Machining without lubrication				

The analysis of variance in Table II shows that the interaction feed and depth of cut clearly influence the temperature (P-value =0.04). After processing, we obtain the prediction equation (2):

$$\textbf{T° tool} = 89.6 + 0.054\ Vc - 638\ a - 19.2\ P - 0.000215\ Vc^2 + 1640\ a^2 + 32.3\ P^2 + 0.572\ Vc\times a + 0.036\ Vc\times P + 368\ a\times P \quad (2)$$

The residual distribution analysis is based on a powerful test for the detection of non-normality.

The normal probability plot is shown in Figure 8. It is clear that the residuals lie close to the reference line, which implies that the errors are normally distributed.

TABLE II. ANALYSIS OF VARIANCE

Source	DF	Adj SS	Adj MS	F-value	P-value
Regression	9	34124.1	3791.57	12.63	0.000
Vc	1	8.0	7.95	0.03	0.871
a	1	677.8	677.83	2.26	0.139
P	1	38.2	38.18	0.13	0.723
Vc×Vc	1	14.4	14.39	0.05	0.828
a×a	1	438.5	438.46	1.46	0.232
P×P	1	260.5	260.46	0.87	0.356
Vc×a	1	116.4	116.40	0.39	0.536
Vc×P	1	17.9	17.85	0.06	0.808
a×P	1	1333.7	1333.65	4.44	0.040
Error	54	16212.6	300.23		
Total	63	50336.7			

TABLE III. COEFFICIENTS OF EQUATION (2)

Term	Coef	SE Coef	T-value	P-value	VIF
Constant	89.6	43.3	2.07	0.043	
Vc	0.054	0.335	0.16	0.871	67.55
a	-638	424	-1.50	0.139	75.61
P	-19.2	53.8	-0.36	0.723	48.28
Vc×Vc	-0.000215	0.000982	-0.22	0.828	53.63
a×a	1640	1357	1.21	0.232	63.50
P×P	32.3	34.7	0.93	0.356	32.25
Vc×a	0.572	0.918	0.62	0.536	17.03
Vc×P	0.036	0.146	0.24	0.808	13.11
a×P	368	175	2.11	0.040	14.92

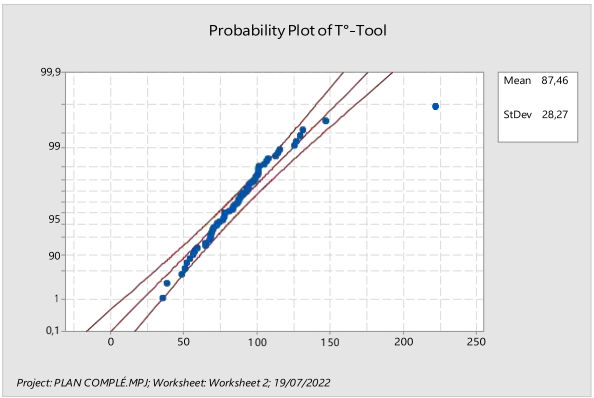

Fig. 8. Normality line of the distribution of residuals.

IV. RESULTS AND DISCUSSION

The test results were divided according to the method of measurement, either in zone 2 or zone 3. The results obtained during the first method (zone 2) have been discussed in [12]. We present the results obtained in zone 3 where the temperature measurement is made just after the release of the tool. The comparison between the experimental values and the predicted values for the obtained temperature model, is represented graphically in Figure 9 for the complete design of the experiment. The latter shows that the experimental and the estimated values are close with an accuracy of 88.87%. These results prove the accuracy of the model. Furthermore, they confirm that the model can be effectively used to predict the

machining temperature with a 95% confidence interval.

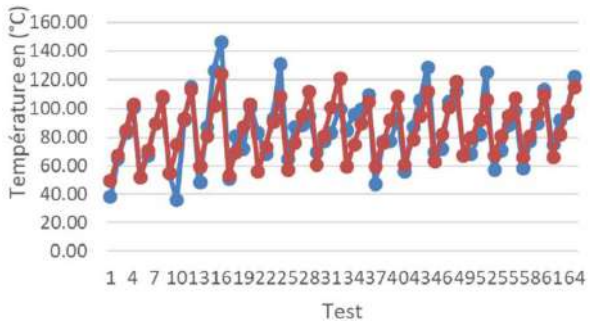

Fig. 9. Experimental and predicted curves

Figure 10 shows the different interactions. The interaction Vc×p (between the cutting speed and the depth of cut) is noticed to have an imposing effect. On the other hand, the interaction

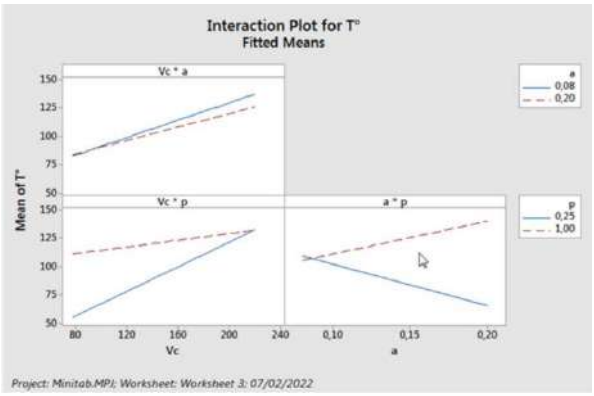

Fig. 10. Interactions of cutting conditions.

a×p (between the feed and the depth of cut) and the interaction Vc×a (between cutting speed and feed) are less significant. Most of the energy released is spent in deforming the chips both cutting and plastically. Part of this energy is transformed into heat by friction at the two interfaces (tool-chip and tool-workpiece). The generation of heat during machining causes the temperature to rise in the process area. It turns out that the cutting speed is the imposing factor on the cutting temperature. To visualize the influence of the cutting parameters on the temperature (T°), the response surfaces (2D) are presented below.

The isotherm in Figure 11 shows that the cutting temperature increases with the feed over the interval of 0.08mm/rev and 0.14mm/rev, with a drop in temperature over the interval 0.14mm/rev and 0.2mm/rev. The thermogram in Figure 12 shows the concentration of heat on the tool tip and chip fragments. The isotherm in Figure 13 shows that the cutting temperature increases with increasing cutting speed and depth of cut. The thermogram in Figure 14 shows the heat distribution on the cut face. The isotherm in Figure 15 shows that the temperature increases for the interval of the feed rate (0.14mm/rev-0.2mm/rev) and the depth of cut (0.75mm-1mm) and it remains moderate for the lower intervals. As can be seen, for a depth of cut equal to 0.25mm, the temperature decreases. The thermogram in Figure 16 shows the concentration of heat on the nose of the tool.

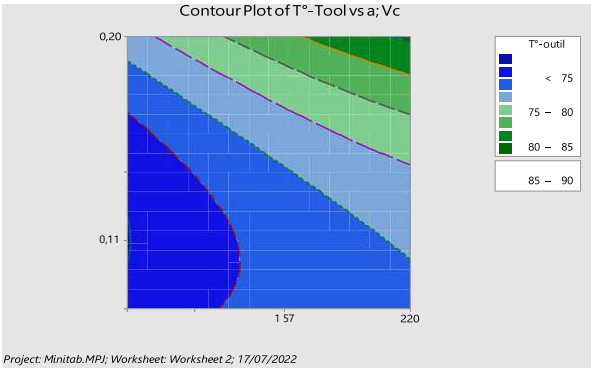

Fig. 11. Temperature isotherm as a function of cutting speed and feed rate

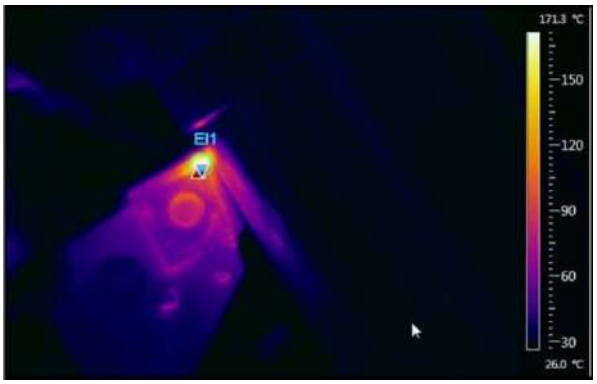

Fig. 12. Thermogram for Vc=78m/min, a=0.08mm/rev, and P=0.25mm

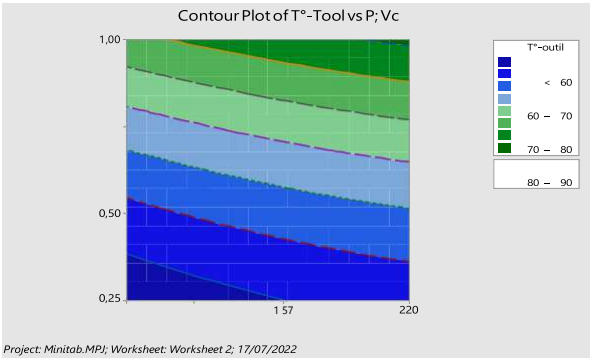

Fig. 13.Isotherm as a function of cutting speed and depth of cut.

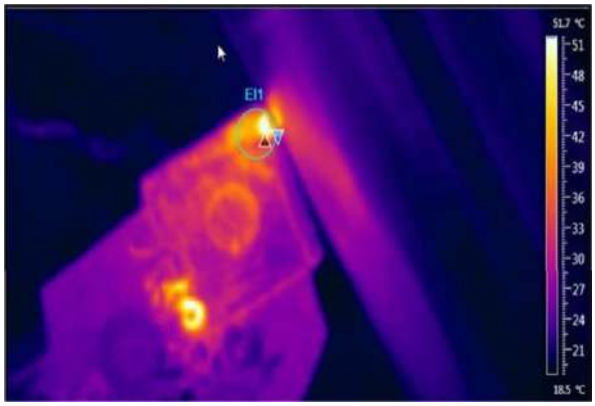

Fig. 14.Thermogram for Vc=157m/min, a=0.08mm/rev, and P=0.75 mm

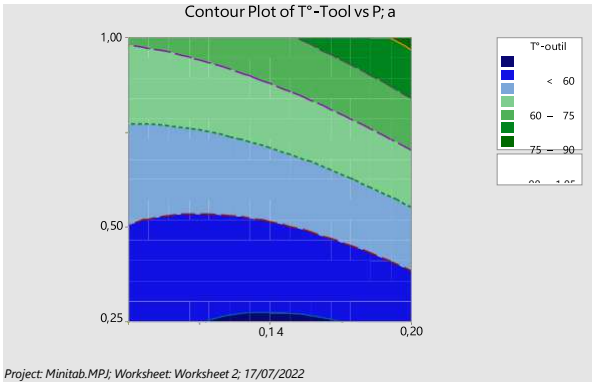

Fig. 15. Isotherm as a function of feed rate and depth of cut.

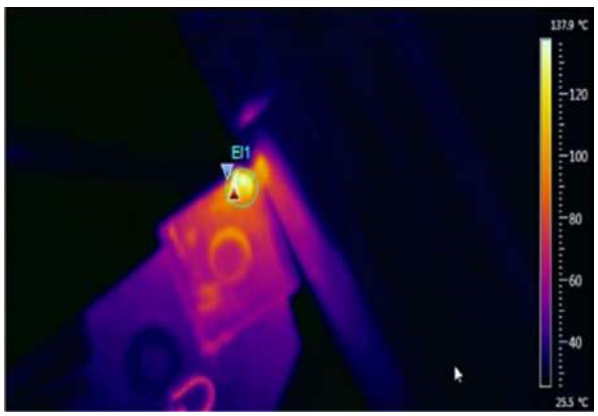

Fig. 16. Thermogram for Vc =112m/min, a=0.14 mm/rev, and P=1 mm.

V. OPTIMIZATION OF CUTTING CONDITIONS

This goal can be achieved by intervening in the different stages of the production of the part:

- Design of the assembly to be manufactured.
- Part definition.
- Choice of manufacturing range.
- Choice of cutting conditions.

Cutting conditions can be optimized by setting an optimum cutting temperature obtained during experimentation. It can be given by the manufacturer depending on the technical characteristics of the pad. In this paper, a method is proposed to find the optimal cutting conditions. The choice of cutting conditions has a direct impact on the machining time of the part, therefore on the cost price of machining and on the productivity improvement. The optimization plot allows us to interactively modify the cut parameters of the input variables to perform sensitivity analysis and possibly improve the initial solution. We apply the optimization technique of the Minitab software used in the current work, for the measurement just after the release of the tool and under the following machining constraints:

$$78 \leq Vc \leq 220; \qquad 0.08 \leq a \leq 0{,}2 ; \quad 025 \leq p \leq 1 \qquad (3)$$

By choosing a temperature of 100° we obtained the results of the optimized input variables grouped in Table IV.

TABLE IV. OPTIMIZED CONDITIONS

T°					
Solution	**Vc**	**a**	**p**	**Fit**	**Desirability**
1	149	0.14	0.348485	99.967	0.999446
2	220	0.185560	0.256348	100.148	0.997301
3	78	0.150854	0.991703	103.348	0.939131

For desirability index equal to 0.999446, the optimal conditions are: Vc=149m/min, a=0.14mm/rev, p=0.34 mm. The optimization diagram in Figure 17 shows the influence of the cutting conditions with the fixed cutting temperature.

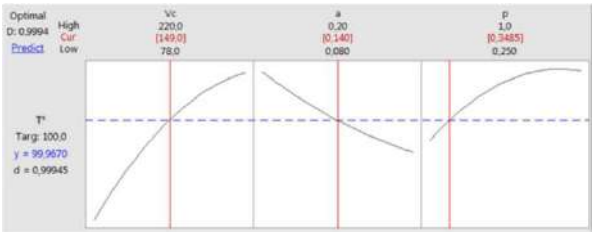

Fig. 17. Impact of cutting conditions.

VI. CONCLUSION

This work shows that a thermography technique is a useful procedure for establishing an experimental methodology for evaluating cutting temperature during the machining operations. It is therefore a method of mapping temperatures in real time, allowing the evaluation over time of surface thermal phenomena. The advances in manufacturing technology require us to estimate and predict the temperature rise at different points of the workpiece-tool interface in order to optimally adjust different parameters beforehand to

improve machinability.

No previous research work has used only an infrared camera. The major difficulty in taking the machining sequence by the camera is the rolling of the chip, hence the need to choose the cutting conditions well and to have good protection and fixing. The experimental results allow us to conclude that the variations of the temperatures are very weak in permanent regimes for invariant cutting parameters, that the stresses applied by the tool on the machined material lead to a rise in temperature and that the heat flow generated during machining is largely evacuated by the chip. The experiments allowed us to determine the temperatures of the machined surface of the part as well as that of the cutting edge for different cutting parameters. The fit of the model was checked and found to be adequate at an accuracy level of 88.87% for temperature measurements just after tool release. This demonstrates that the model can be used to predict the cutting temperature. The technique could be an economical and effective method for temperature prediction and optimization of the cutting parameters.

The quality of the parts produced depends largely on the cutting conditions and this quality is an important parameter during the production of mechanical parts. The cutting fluid is also an important parameter since it allows to limit the rise in temperature in the cutting zones and to establish the presence of a fluid film between the different surfaces. However, in order to protect the environment and the health of the personnel, the current trend is to limit and even eliminate the use of cutting fluids

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SCIENTIFIC PROGRAM

ISCDFAMS 2023 SYMPOSIUM PROGRAMME

FIRST DAY 14 November, 2023

14.11.2023 Tuesday Türkiye İstanbul Time	Hall 1	Hall 2	Hall 3
	OPENING CEREMONY (Ankara SALOON) Chair: Ayşenur Uçar		
08:45-08:50	National Anthem and Respectful Silence		
08:50-09:00	Prof. Dr. Kürşat AKBULUT, Atatürk University - Speaker on behalf of the organizing committee (**Türkiye İstanbul Time-Zoom**)		
Türkiye İstanbul Time 09:00-10:30	(09:00-09:10) Prof. Dr. Yavuz DEMİR - Ankara Bilim University - Rector (09:10-09:20) Prof. Dr. Atilla KESKİN - Atatürk University - Vice Rector (09:20-09:30) Prof. Dr. Necdet ÜNÜVAR, Ankara University - Rector (09:30-09:40) Prof. Dr. Musa YILDIZ, Gazi University - Rector (09:40-09:50) Prof. Dr. Bülent ÇAKMAK, Erzurum Technical University - Rector (09:50-10:00) Prof. Dr. Akın LEVENT, Erzincan Binali Yıldırım University - Rector (10:00-10:10) Prof. Dr. Abdulhalik KARABULUT, Ağrı İbrahim Çeçen University - Rector (10:10-10:20) Prof. Dr. Halil İbrahim ZEYBEK, Gümüşhane University - Rector (10:20-10:30) Prof. Dr. Ali Cengiz KÖSEOĞLU, Ankara Yıldırım Beyazıt University - Rector		
10:30-11:00	**COCKTAIL**		
11:00-11:30	**Keynote Speech:** ***Prof. Dr. Bayram ŞAHİN (Ege University – Türkiye)*** **(Ankara SALOON) Chair: Ayşenur Uçar** *On geometric and physical interpretations of basic concepts on a (semi-Riemann) manifold (Türkiye İstanbul Time-Zoom)*		
11:30-12:00	**Invited Speakers:** ***Prof. Dr. İsmail Naci CANGÜL (Uludağ University-Türkiye)*** **(Ankara SALOON) Chair: Ayşenur Uçar** *Omega invariant and Realizability of Graphs (Türkiye İstanbul Time-Zoom)*		
12.00-13.00	**LUNCH (Self Provided)**		
13:00-13:30	**Invited Speakers:** ***Prof. Dr. James Peters (University of Manitoba - Canada) Chair: Ayşenur Uçar*** *Self-Symmetries in Descriptively Near Envelopes* **(Amplifier 1, Amplifier 2, Amplifier 3) (Türkiye İstanbul Time-Zoom)**		
13:30-13:45	**COFFEE BREAK**		

14.11.2023 Tuesday	Hall 1	Hall 2	Hall 3
13:45-14:00	Chair: Ahmed Yiğider (Co-Chair: Davaez Qasi)	On the bicubic Bezier surfaces and paraboloids in E³ **Şeyda Kılıçoğlu**	The Indexs For Ditopological Texture Graphs **Tuğçe Kunduracı**
14:00-14:15		On Lie Differentiability Conditions of Some Polynomial Structures in the Semi Tangent Bundle **Imran Gunes**	
14:15-14:30		Short Notes for Tangent Indicatrix and Binormal Indicatrix **Ayşenur Uçar**	Quaternions and Three-Dimensional Rotation **Hicran Aşar**
14:30-14:45	Chair: Mutlay	A modelling of the natural logarithm and Mercator series as 5^(th),6^(th),7^(th) order Bezier curve plane **Şeyda Kılıçoğlu**	Some Tensor Structures On Semi-Tangent Bundle t(M) **Mustafa Kır**
14:45-15:00	**COFFEE BREAK**		
15:00-15:15	Chair: Havva Turan (Co-Chair: Ayşenur Uçar)	Diagonal Lift Problems in a Special Class of Semi-Cotangent Bundle **Hacer Cengiz**	An Enhanced Mechanism for Boosting Salp Swarm Algorithm **Meriem Nacer**
15:15-15:30	On the convergence rate of robin type overlapping domain decomposition methods **Ahmed Salah Chibi**	Some Horizontal Lift Problems in the Semi-tensor Bundle **Merve Aydın**	
15:30-15:45	New inequalities of matrix means for sectorial matrices **Yassine Bedrani**	Covariant Derivatives of Structures on Semi Tangent Bundle **Kübra Atasever**	New stability number of the Lord-Shulman system with microtemperature effects **Marwa Boudellou**
15:45-16:00	Acceleration Slant Curve in ${E_1}^3$ **Hasan Altınbaş**	Pseudo-Riemannian Metric and Lorentzian Almost r-Paracontact Structure On The Semi-Tangent Bundle **Gülnur Çağlar**	Existence Theory Of L^1-Solutions For Fractional Differential Equations **Abdelatif Boutiara**
16:00-16:15	**COFFEE BREAK**		
16.15-16.30	Chair: Mohammed Yiğider (C	Chair: Yassine Bedrani (Co-Chair: Mustafa Özkan)	Chair: Marwa Boudellou (Co-Chair: Gökhan Mumcu)
16.30-16.45			
16.45-17.00			
17.00-17.15			
18:00-20:00	**DINNER (Traditional Erzurum Cag Kebab)-(ESAV)-14 November, 2023 (18:00-20:00)**		

SECOND DAY 15 November, 2023

09.00-09.30	**Invited Speakers: Prof. Dr. Arif SALIMOV (Bakü University – Azerbaijan) (Ankara SALOON)** *A new lift construction of connections in the bundle of 2-jets* **(Türkiye İstanbul Time-Zoom)**
09:30-09:45	**COFFEE BREAK**

15.11.2023 Wednesday	Hall 1	Hall 2	Hall 3
09.45-10.00	Hyperbolic Pell Polynomials and their some Properties	The locally trivial bundle above the complex projective line	Almost contact metric manifoldswith certain condition

	Chair: Tuğçe Kunduracı (Co-chair:Ali Çakmak)	Havva Turan	Chair: Çağrı Karaman (Co-chair:Rabiaa Aouaiti)	Irina Sorokovnina	Chair: Irmak Uzun Bayar (Co-chair:Ümmü Şahin Şener)	Benaoumeur Bayour
10.00-10.15				Pointwise bi-slant Riemannian maps into Hermitian manifolds **Yılmaz Gündüzalp**		Approximate Analytical and Numerical Solutions of Fractional Black-Scholes Option Pricing Equations **Ummü Şahin Şener**
10.15-10.30		Short Time Behaviour of a Three-dimensional Dam-break Flow Involving Two Fluids with Free Surfaces **Tuğba Yazan**				Numerical modeling of the mechanical behavior and repair of a piplines by composite sleeve **Sid Ahmed Benmansour**
10.30-10.45						Numerical Analysis of a Non-Linear Stochastic Epidemic Model **Ali Raza**
10.45-11.00	**COFFEE BREAK**					
11.00-11.15	Chair: Berrin Şentürk (co-Chair:Bayram Şahin)	Study of Structural ,dynamic and mechanical properties of Hf2CO2 Mxene,ab-initio calculations **Chewki Ougherrb**	Chair: Bilel Neggal (co-chair: Olga Belova)		Poster	
11.15-11.30				Stability of generalized P-harmonic maps **Ahmed Mohammed Cherif**		
11.30-11.45				A determination of effective moisture diffusivity during solar dryer of tomato slices **Korti Abdel Illah Nabil**		Expansions of q-opretional equations for q-polynomials and related problems of q-PDE **Jian Cao**
11.45-12.00		Existence result for Elliptic equation on Riemannian manifolds **Imad Amrani Zerrifi**		Fundamental equations on conformal Fedosovov spaces **Chakibek Almazbekov**		Cutting Parameter Optimization based on Online Temperature Measurements (Poster) **Djamal Kara Ali**
12.00-13.00	**LUNCH (Self Provided)**					
13:00-13:30	**Invited Spiker : Prof. Dr. Josef Mikes (Palacky University-Czech Republic) (Ankara SALOON)** *Geodesic mapping and their generalization* **(Türkiye Istanbul Time-Zoom)**					
14.00-14.40	**COFFEE BREAK**					
15.00-15.15	an (co-chair:Karima b)	Regularized Sinc Collocation Method Applied to Fredholm Integral Equation of the first kind **Bilel Neggal**	an)		uğ co-chair:Mesut	
15.15-15.30		On a new non-linear coupled system involving two types of fractional derivatives **Abdelkader Amara**				On a generalized degenerate fractional differential IVP in a Banach space **Said Mazouzi**

15.30-15.45	Chair: Önder Erkas D		Chair: Djamal Kara Tu		Chair: Sezai Kızılt Altı	Data-Oriented Bayesian Network Model of Breast Cancer Risks **Ersin Şener**
15.45-16.00		Fuzzy Double-Indexed Norm Space in Terms of a-Level set **Sevda Atapınar**		Characterization of strong invariance with respect to diferential inclusions with time-dependent maximal monotone operators **Karima Dib**		Characterization of Special Space Curves with Euler Angles **Mesut Altınok**
16.00-16.15	**COFFEE BREAK**					
16.15-16.30	Chair: Ali Çakmak(co-chair:Seyda Öztürk)		Chair: Sait Taş (co-chair:Kaddour Zegga)		Chair: Nejmi Cengiz(co-chair: Tülay Yıldırım)	
16.30-16.45						
16.45-17.00						
17.00-17.15						
18:00-20:00	**GALA DINNER (Beyoğlu Halimbey Restaurant)-15 November, 2023 (18:00-20:00)**					
THIRD DAY 16 November, 2023						
9:00-09:30	**Invited Speakers:** ***Prof. Dr. Tony SHANNON (University of Technology Sydney– Australia)*** **(Ankara SALOON)** *Some Conjectures About Number Theory Triangles* **(Türkiye Istanbul Time-Zoom)-(Chair: Necati Can Açıkgöz)**					
16.11.2023 Thursday		**Hall 1**		**Hall 2**		**Hall 3**
10.00-10.15	Chair: Pelin Karaca Kalkan (Co-Chair: Dawez Qasi		Chair: Mustafa Özkan (Co-Chair:	New hybrid conjugate gradient method with descent condition and globally convergent **Amina Hallal**	Chair: Muhammed Yiğider (Co-	A mathematical model in epidemiology **Khadidja Hamli**
10.15-10.30		Existence Results for Second order Functional Differential Inclusions **Medjadj Imene**		Synthesis of dichlorodiazabutadienes based on tert-butyl benzaldehyde **Gulnar Atakishiyeva**		
10.30-10.45		Primal-Dual Interior Point Methods for SDLCP Based on a New Type of Kernel Functions **Nesrine Tabchouche**		Expansions of q-operational equations for q polynomials and related problems of q-PDE **Jian Cao**		On a New Combination of Orthogonal Polynomials Sequences **Mohammed Cherif Bouras**
10.45-11.00		Graph Energies Emerging from Graph Ricci Curvatures **Sibel Paşalı Atmaca**				Existence of positive solutions for a weighted quasilinear elliptic system **Yamina Hamzaoui**
11.00-11.15	**COFFEE BREAK**					

11.15-11.30	Chair: Ali Ulvi Özgül (co-chair: Sevda Öztürk)	The Geometry of a non-conformal deformation of a metric and bi-harmonicity **Kaddour Zegga**	Chair: Sezai Kızıltuğ (co-chair: Kübra Karataş Selam)		Chair: Ali Çakmak (co-chair: Erdoğan Özbayrak)	DNA codes Over The Ring Z_4[Z_4+uZ_4)[Z_4+uZ_4(u^2Z)_4 **Tülay Yıldırım**
11.30-11.45		On Ricci pseudosymmetric Kahler spaces **Patric Peska**		A contribution to the thermal field evaluation at the tool-part interface if machining conditions **Nasr-edinne Benhadji Serradj**		A new sequential differential problem with Caputo-Hadamard fractional derivatives **Amira Abdelnebi**
11.45-12.00		Sizing of a drip irrigation system with photovoltaic pumping **Nasr-edinne Benhadji Serradj**				
12.00-12.15		Permanents and Determinants of some Hessenberg Matrices with Generalized Jacobsthal-Like Sequences **Kübra Karataş Selam**				Quaternionic Normal-Direction Curves and Applications **Erdoğan Özbayrak**
12.00-12.15	**COFFEE BREAK**					
12.20-13.00	**LUNCH (Self Provided)**					
13:00-13:30	**Invited Speakers: *Prof. Dr. Yusuf Yaylı (Ankara University - Turkey)*** **(Ankara SALOON)** Approximations of *Approximations of Surfaces Along Curves* **(Türkiye Istanbul Time-Zoom) (Chair: Necati Can Açıkgöz)**					
13.30-15.00	**COFFEE BREAK**					

FOURTH DAY 17 November, 2023 (ONLINE)-Türkiye Istanbul Time-Zoom

09:00-09:30	***Invited Speakers: Prof. Dr. Marin MARIN (Transilvania University of Brasov– Romania)*** **(Ankara SALOON)** *On Deformation on a Cosserat Thermoelastic Body with Microtemperatures and Inner Structure*- **Türkiye Istanbul Time-Zoom-(Chair: Hatice Seda Yılmaz)**					
17.11.2023 Friday		**Hall 1**		**Hall 2**		**Hall 3**
09.30-09.45	Chair: Hatıra Sağır (Co-Chair: Arzu Akbulut)	Impulsive generalized proportional Riemann-Liouville fractional differential equations: Stability concepts **Snezhana Hristova**	Chair: Şule Şener Kılıç (Co-Chair: Elif Demirağ)	Investigation Of The Unsteady Three-Dimensional Magnetohydrodynamic Flow Of Nanofluids Over A Decelerated Rough Rotating Disk. **ELİF DEMİRAĞ**	Chair: Meher Aslancı (Co-Chair: İbrahim Şanlıbaba)	Real Lines on Cubic Surfaces **Mecid Sariye**
09.45-10.00		Sound Wave Propagation along a Coaxial Duct with a Step Discontinuity via Wiener-Hopf Method **Adile Uluca**		Some Inequalities For The Generalized Digamma Function **Emrah YILDIRIM**		On the Robustness of the Trajectories of Control Systems with Respect to Fast Consumption of the Control Resource **Khalik Guseinov**
10.00-10.15		Existence and nuiqueness results of a Lasota-Wazewska model with an iterative term		A Numerical Approach of a Singularly Perturbed Differential-Difference Equation with Mixed Type		Regional controllability of fractional-order dynamical system with time-varying delays in the state

	Chair: F	**Marwa Khemis**	Chair:	**Erkan Cimen**	Chair: S	**Wadii Ghandor**
10.15-10.30		Examination of the Existence of Coding on Violin Strings **Bahar Kuloğlu**		Some bounds for the topological indices on fuzzy graphs **Ezgi Kaya**		Properties and applications of bipolar fuzzy sets **İbrahim Şanlıbaba**
10.30-10.45	Chair: Tülay Erişir (Co-Chair: Özen Özer)	On Investigating Numbers in the Sets of Diophantine D(∓2): New Insights **Özen ÖZER**	Chair: Mesut Karabacak (Co-Chair: Melih İs)	A new collocation method for solving fractional Riccati equation **Fatemeh Soleyman**	Chair: Arzu Akbulut (Co-Chair: Murat Boduri)	Investigation of Power-Law Fluids over a Decelerated Rotating Disk **Serkan Ayan**
10.45-11.00		The Role of Upper Triangular Matrices in Secure Communication with Examples **Özen ÖZER**		Hyperbolic-hyperbolic singular perturbation for aquasilinear problem with a nonlinear dissipative term **Sara MOKHTARI**		Some Approximation Properties of Lupaş-Jain Operators **Murat Bodur**
11.00-11.15		Phase Type Distribution Queueing Systems **Lounes Ameur**		Is the shortest path always the fastest? **Fatmanur Aydoğmuş**		On Nez Characterizations Curves in 5-Dimensional Lorentzian Space **Mustafa KAYMAZ**
11.15-11.30		Teaching With Dynamic Mathematics Software Geogebra: Examining Postgraduate Theses **Ebru KORKMAZ**		Some Algebraic Approaches in Nearness Theory **Melih İs**		On the generators of F_2F_4-skew cyclic codes **Mustafa Sarı**
11.30-13.30	**LUNCH (Self Provided)**					
13.30-14.00	**Invited Speakers: Prof. Dr. Praveen AGARWAL (Anand International College of Engineering, Jaipur– India)**					
	Certain Saigo Type Fractional Integral Inequalities and Their q-Analogues **(Ankara SALOON) Türkiye Istanbul Time-Zoom-(Chair: Merve Aydin)**					
17.11.2023 Friday		**Hall 1**		**Hall 2**		**Hall 3**
14.30-14.45	Chair: İsa Yıldırım (Co-Chair: Osman Avcıoğlu)	The Exact Solutions of the Nonlinear Landau-Ginzburg-Higgs Equation **Özlem OKUYAN**	Chair: Özen Özer (Co-Chair: Neriman Kartal)		Chair: Derya Sağlam (Co-Chair: Faik Babadağ)	Exact Traveling Wave Solutions for the non-linear couple Drinfeld-Sokolov-Wilson (DSW) Dynamical System Using Extended Jacobi Elliptic Function Expansion Method **Nisa ÇELİK**
14.45-15.00		On the mock-Chebychev Polynomial Interpolation **Nisa Nur Çakır**		Discrete Fractional Order Economic Model in a Barabasi-Albert graph with N = 10 nodes **Neriman Kartal**		Dual vectors with Jacobsthal and Jacobsthal-Lucas number sequences **Faik Babadağ**
15.00-15.15		The Category of Groups Ẑn **Osman Avcıoğlu**		An Approximate Solution for System of Delay Differential Equations with Delay Parameter **Sevket Uncu**		Dual Number Sequences: Quantum Calculus Approach with Applications **Faik Babadağ**
15.15-15.30		Split (pre)crossed modules over R-algebroids **Osman Avcıoğlu**		Flat Approximations of Hypersurfaces **Çağla Ramis İlguz**		Shehu Variational Iteration Method for Solutions of Schrödinger Equations **Ece Güneş Zaralı**

15.30-15.45	Ö	On Selective Covering Properties in Bitopological Spaces **Necati Can Açıkgöz**		Enhancing Music Streaming with Spotify's Algorithm: A Deep Dive into Recommendation Systems **Zeynep Budak YILMAZ**	Ö	Approximation by nonlinear Szász-Mirakyan operators of Durrmeyertype **Sevilay Kırcı Serenbay**
15.45-16.00				**COFFEE BREAK**		
17.11.2023 Friday		**Hall 1**		**Hall 2**		**Hall 3**
16.00-16.15	**Chair: Olgun Durmaz (Co-Chair: Ömer Faruk**	Schatten Class Generalized Toeplitz Operators on Harmonic Bergman-Besov spaces of the Unit Ball **Ömer Faruk DOĞAN**	**Chair: Ebru Korkmaz (Co-Chair: Zeynep Özkurt)**	Tame and Wild Automorphisms on Free Leibniz Algebras **Zeynep Özkurt**	**Chair: Arzu Akbulut (Co-Chair: Döndü Öz)**	Some results on the existence of positive solutions for the iterative system of the impulsive boundary value problem **Döndü Öz**
16.15-16.30		Biharmonic Sections **Bouazza Kacimi**		On the Rank of an Element in Free Leibniz Algebras **Zeynep Özkurt**		A New Numerical Method to Solution of Conformable Fractional Partial Differential Equation **Hakkı Güngör**
16.30-16.45		Some Properties of Developable Surfaces **Çağla Ramis İlgüz**		An Inverse Source Problem for the Quantum Kinetic Equation **Fikret Gölgeleyen**		On New Banach Sequence Spaces e_0^(a,b) (u,p,q) and e_c^(a,b) (u,p,q) and Their duals **Damla BARLAK**
16.45-17.00		Commutators generated by Calderon-Zygmund operators with kernelsof Dini's type and weighted Lipschitz functions on generalized weighted Morreyspace **Ferit Gürbüz**		Numerical Study of a New Time-Fractional Monkeypox Model using Caputo Fractional Derivatives **Shyamsunder Kumawat**		An Exact Solutions of the combined KdV-mKdV Equation by the Consistent Riccati Expansion Method **Arzu Akbulut**
17.00-17.15				**COFFEE BREAK**		
17.11.2023 Friday		**Hall 1**		**Hall 2**		**Hall 3**
17.15-17.30		Phase Type Distribution Queues **Lounes Ameur**		Existence Result for Elliptic Equation on Riemannian manifolds **Imad Amrani Zerrifi**		Ulam-type Stability for implicit fractional proportional equation **Rabiaa Aouafi**
17.30-17.45		On some recursive methods in matrices operations **Youssouf Mezzar**		Physics Informed Neural Network Method for the Numerical Solution of Benney-Roskes/Zakharov-Rubenchik System **Burcu Ece Alp**		Some Characterizations on Hypersurfaces of Riemannian Product Manifolds **Mehmet Gülbahar**

17.45-18.00	**Chair: Harun Selvitopi(Co-Chair: Lounes Ameur)**	Approximate Solutions of Schrödinger Equations by Laplace Variational Iteration Method **Arzu Güleroğlu**	**Chair: Olgun Durmaz (co-Chair: Burcu Ece Alp)**	Algebraic and Differential Identities on Rings and Algebras **Faiza Shujat**	**Chair: Nazlı Karaca (Co-Chair: Rabiaa Aouafi)**	Study of nonlocal-delay conformable fractional differential systems of second order in Banach spaces **M.Hachem BEN TAHIR**
18.00-18.15		Investigation of stresses occuring in a circular disk consisting of different annular materials by mathematic modeling **Hüseyin Fırat Kayıran**				New Jacobi wavelets' kernel for Support Vector Machines **Nadira Abbassa**
18.15-18.30		Existence Results for Second order Functional Differential Inclusions **Imene Medjadj**		Ulam-type Stability for implicit two-order fractional Integro-Differential equation with 3-point boundary conditions **Nasrallah İlhem**		Positive Solutions of a Fractional Order p-Laplacian Boundary Value Problem **Gül Gizem Yüce**
18.30-18.45		Obstacle Two-phase Problems with Hardy Potential **Ahmed Abergi**		Horizontal lift of affine connection to the (0,2) type tensor frames **Habil Fattayev**		The Study of Pre-Service Mathematics Teachers' Approach on Students' Misunderstanding For Four Possible Answers to Solve a problem **Manouchehr Behboudi Asl**
18.45-19.15		The deformation of connections in cocongruence K_(n-m)m **Olga Belova** Some notes on Clairaut conformal Riemannian submersions **Murat POLAT**		Reaction- Diffusion SIR Model and an Exact Solution **Sevda ÖZTÜRK** On a New Class of Hypergeometric Function **Praveen Agarwal**		A note Hopf's conjecture **Sergey Stepanov**
19.15-19.30		A study on Conformal semi-invariant Riemannian maps to Sasakian manifolds **Sümeyye KARAGÖL**				Bioconvention near a strechable boundary with convective conditions **Touseef Fayaz**
19.30-19.45		A variational approach to a constrained hyperbolic Kirchoff type problem **Fatima Ezahra Bentata**				Apollonius Problem and Caustics of an Ellipsoid **Yagub Aliyev**
19.45-20.00						
20.00-20.15						